STUDIA METODOLOGICZNE
NR 35 • 2015, 79-96
DOI: 10.14746/sm.2015.35.6

JOSÉ MARTÍN CASTRO-MANZANO

Słupecki's Rule for Diagrammatic Reasoning

ABSTRACT: In this contribution we pursue a simple goal: to show that Słupecki's rule can be applied to a diagrammatic interpretation of syllogistic. With this goal we expect to bring more attention on Słupecki's rule and suggest that diagrammatic systems are, by no means, irrational approaches to logic.

KEY WORDS: diagrammatic logic, syllogistic, $L_{\Box}$, decidability

1. Introduction

According to Woleński and Zygmunt, when Jerzy Słupecki (1904-1987) passed away, those who witnessed his departure very well could have the impression that the Warsaw school of logic ceased to exist too: Słupecki was the last Warsaw logician alive who began his scientific career in the golden years of Polish logic [Woleński & Zygmunt, 1989]. His contributions were many and substantial: from many-valued logics to didactics of logic, his interests were wide and they included a particular one in syllogistic, just as his thesis advisor, Jan Łukasiewicz.

Within Slupecki's advances to syllogistic we find a particular rule that bears his name, a rule that allowed him to make some discoveries that, in the opinion of Łukasiewicz himself, were the most important in syllogistic since Aristotle [Łukasiewicz, 1970]. In this contribution we pursue a simple goal: to show that Słupecki's rule can be applied to a diagrammatic interpretation of syllogistic. With this goal we expect to bring more attention on Słupecki's rule (SR from now on) and suggest that diagrammatic systems are, by no means, irrational approaches to logic in general, and syllogistic in particular, for such systems provide equivalent logical procedures however different from traditional sentential approaches.

To reach our goal we have organized this paper in the next way. In Section 2 we begin with a brief exposition of SR. In Section 3 we develop some ideas behind the notion of diagrammatic logical consequence. Then, in Section 4, we explain some general aspects of a diagrammatic system and, after that, we propose how SR can be applied to it.

2. Słupecki's rule

In a controversial and groundbreaking work, Łukasiewicz [1951] showed that Aristotelian syllogistic could be axiomatically developed on the grounds of standard propositional calculus, substitution and detachment rules (for asserted and rejected expressions), and some axioms of assertion and rejection. His axiomatization can be summarized in Table 1.[1]

Table 1. Łukasiewicz's axiomatization for Aristotelian syllogistic

Axioms		Rules
Assertion	Uaa Iaa CKUcbUacUab CKUcbIcaIab	Substitution: If α is an asserted expression, then any expression produced from α by a valid substitution is also an asserted expression. Detachment: If $C\alpha\beta$ and α are asserted expressions, then β is an asserted expression.

[1] A *categorical proposition* is a proposition composed by two terms, a quantity, and a quality. The subject and the predicate of a proposition are called *terms* and while the term-schema a denotes the subject term of the proposition, the term-schema b denotes the predicate. Following Łukasiewicz's notation, the *quantity* may be either universal (U) or particular (I). And finally, the *quality* may be either affirmative or negative (N). With this notation the four categorical propositions are represented by Uab (All a is b), NUab (Some a is not b), Iab (Some a is b), NIab (No a is b). C and K stand for implication and conjunction, respectively.

Rejection	CKUbcUacIab CKNIbcNIacIab	Substitution: If β is a substitution of α, and β is rejected, α must be rejected too. Detachment: If the implication Cαβ is asserted, but β is rejected, α must be rejected too.

From the axioms and rules of assertion it is possible to derive the fundamental tenets of Aristotelian syllogistic: the square of opposition and all the valid syllogisms. On the other hand, from the axioms and rules of rejection all the invalid moods of syllogistic can be rejected. However, according to Łukasiewicz [1951], this axiomatic approach does not suffice to describe Aristotelian syllogistic because:

> […] there exist significant expressions, for instance CIabCNAabAba, which can neither be proved by our axioms and rules of assertion nor disproved by our axioms and rules of rejection. I call such expressions undecidable with respect to our basis. Undecidable expressions may be either true in the Aristotelian logic or false. The expression CIabCNAabAba is, of course, false. [p. 100][2]

Under this context of (un)decidability, Łukasiewicz posed two problems: *i*) is the number of undecidable expressions finite? And *ii*) is it possible to complete the axiomatic system described in Table 1 so that we can decide whether a given expression has to be asserted or rejected? In *On Aristotelian Syllogistic*, Słupecki [1951] answered both questions: the former, negatively (§7, Theorem III); the latter, affirmatively (§9, Theorem V). The rule that bears his name is a result of answering these questions.

In §2, Definition V, Słupecki describes what a rejected expression is. A *rejected expression* is a meaningful expression of Aristotelian syllogistic that is rejected w.r.t. the expression:

`CKUbcUacIab`

Then, according to §8, Definition XIV, a *k-range rejected expression* is defined as follows: let *k* be any natural number, α_1 and α_2 any general-

[2] In this excerpt, the expression CIabCNAabAba must be read as CIabNUabUba.

or particular-affirmative expressions, and *x* any simple expression; then a *k*-range rejected expression is, for *k*=1, any meaningful expression of Aristotelian syllogistic which is a rejected expression according to Definition V, and also any expression having the form:

$$\text{(A) } \mathsf{C}{*}\mathsf{K}{*}\mathsf{N}{*}\alpha_1\mathsf{N}{*}\alpha_2 x$$

provided the expressions

$$\text{(B) } \mathsf{C}{*}\mathsf{N}{*}\alpha_1 x \text{ and (C) } \mathsf{C}{*}\mathsf{N}{*}\alpha_2 x$$

are rejected according to Definition V; and for *k*>1, any meaningful expression which is rejected w.r.t. any set of meaningful expressions, every member of which is a rejected of a lower range than *k*, and also any expressions (B) and (C) are rejected expressions of ranges lower than *k*.

With the aid of these definitions—and of course, other results that would require more space than we have—, Słupecki developed a rule that today bears his name (§9, Proposition III): if the expressions (B) and (C) are both false, then expression (A) is also false; formally:

$$\text{(SR) If } {*}\mathsf{C}\alpha\gamma \text{ and } {*}\mathsf{C}\beta\gamma\text{, then } {*}\mathsf{C}\alpha\mathsf{C}\beta\gamma,$$

that is to say, that if α and β are simple negative expressions and γ is an elementary expression, then if $\mathsf{C}\alpha\gamma$ and $\mathsf{C}\beta\gamma$ are rejected, then $\mathsf{C}\alpha\mathsf{C}\beta\gamma$ must be rejected too (* represents the notion of rejection).

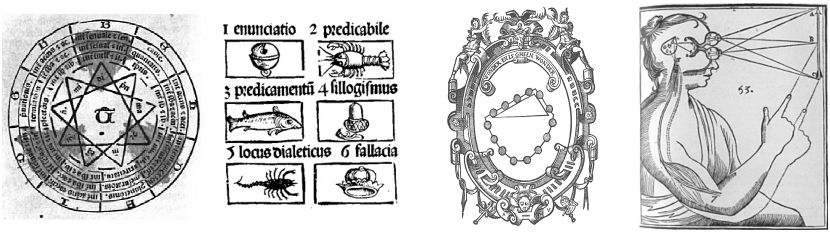
When the axiomatization for Aristotelian syllogistic showed in Table 1 is modified by replacing the second axiom of rejection with SR, Aristotelian syllogistic can be proved to be complete and decidable. We think it is possible to do something similar for a diagrammatic system capable of representing syllogistic.

3. Diagrams and diagrammatic logical consequence

But before we approach such possibility, we would like to dedicate some time to argue that diagrammatic logical systems can be defined in a formal fashion and that we can describe a well behaved notion of diagrammatic logical consequence between diagrams; and in order to do that, we would like to introduce diagrams by paying attention to their expressive power.

This expressive power is something pop culture already recognizes: the XIXth proverb “a picture is worth 10000 words” is quite representative in this sense; but this acknowledgement is much older, for notable examples of confidence in this power can be found in different historical periods. The diagrams of the square of opposition, usually attributed to Apuleius [Londey & Johanson, 1987, p. 109], and the diagrams for syllogisms imputed to Ammonius Hermiae or Philoponus [Hamilton, 1866, p. 420], are some initial examples; however, Ramon Llull (1232-1315) arguably provides the most famous example: he developed *Ars Magna*, a diagrammatic device used to *explain* divine nature to those unable to understand God’s, as if diagrammatic methods were more convincing or expressive than sentential representations (Figure 1a). Thomas Murner (1475-1537) used diagrams in his *Logica Memorativa* in order to *teach* logic (Figure 1b). Dutch mathematician and philosopher of science Simon Stevin (1548-1620) developed another remarkable diagram in his *demonstration* that the efficiency of the inclined plane is a logical consequence of the impossibility of perpetual motion (Figure 1c).[3] And of course, we also have Descartes (1596-1650), who made good use of diagrams in order to *model* hypothesis, such as the mechanics of the pineal gland (Figure 1d).

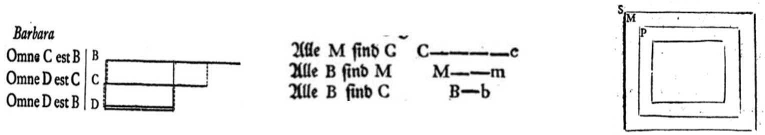
[3] In this second diagram we can appreciate that the chain must be stationary; if the chain were heavier on one side, it would move to such side, but then the chain would move perpetually, which is absurd. Therefore, the chain must remain stationary, and so, the fragment of the chain that is hanging must form a symmetric catenary. Thus, if we cut both sides of the hanging fragment the equilibrium is preserved. But then the part of the chain in the longer side of the triangle is in equilibrium with the fragment of the chain in the triangle’s shorter side: their respective numbers are in inverse ratio to the sin of the angles. Therefore, the mechanical advantage of the inclined plane is proportional to the length of the slope. It is not surprising, thus, that Stevin himself ordered to add the inscription: *Wonder, en is gheen wonder*; wonderful, but not unexplainable.



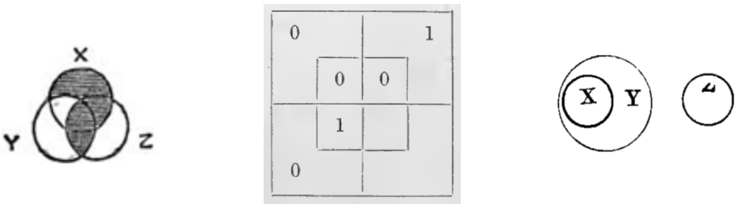
a. *Ars Magna* [Llull, 1501]
a. *Logica Memorativa* [Murner, 1509]
c. *De weedgaet* [Stevin, 1586]
d. *Principles of Philosophy* [Descartes, 1982]

Fig. 1. Notable examples of diagrams for aiding reasoning

What we want to stress is that the expressive power of diagrams for aiding *reasoning* is not news, and specially for syllogistic, as we can see from some notable examples (Figure 2).



a. `CKUmpUsmUsp` in [Leibniz, 1903]
b. `CKUmpUsmUsp` in [Lambert, 1764]
c. `CKUmpUsmUsp` in [Bök, 1766]



d. `CKNUmpUsmNIsp` in [Venn, 1880]
e. `CKUmpUsmUsp` in [Carroll, 1887]
f. `CKNUmpUsmNIsp` [Venn, 1880]

Fig. 2. Notable examples of diagrammatic representations of syllogistic

This confidence in the power of diagrams is understandable. In order to represent knowledge we use internal and external representations. Internal representations convey mental images, for example; while external representations include physical objects on paper, on blackboards, or computer screens. Following [Larkin & Simon, 1987] external representations can be divided into two classes: sentential and diagrammatic.

Sentential representations are sequences of sentences in a particular language. Diagrammatic representations are sequences of diagrams that contain information stored at one particular *locus* in a diagrammatic configuration, including information about relations with the adjacent *loci*; and *diagrams* are information graphics[4] that index information by location on a plane [Larkin & Simon, 1987]. In particular, logical diagrams are two-dimensional geometric figures with spatial relations that are isomorphic with the structure of logical statements [Gardner, 1958, p. 28]. The difference between diagrammatic and sentential representations is that, due to this spatial feature, the former preserve explicitly information about topological relations, while the latter do not—they may, of course, preserve other kinds of relations. This spatial feature provides some computational advantages: diagrams group together information avoiding large amounts of search, they automatically support a large number of perceptual inferences, and they grant the possibility of applying operational constraints (like *free rides* and *overdetermined alternatives* [Shimojima, 1996]) to allow the automation of perceptual inference [Larkin & Simon, 1987].

However, despite this confidence, when it comes to reasoning there is a bias (a tradition?) that supports the claim that while proof-based reasoning

---

[4] Information graphics can be divided into the next classes [Nakatsu, 2009]: quantitative charts (bar-column charts, line graphs, XY scatterplots, pie charts), maps (directional maps, topographic maps, contour maps, weather maps), tables (one way tables, two ways tables, multiway tables), pictorial illustrations, and diagrams, which we can use to study system topology (conceptual models, network diagrams), sequence and flow (flowcharts, activity diagrams), hierarchy-classification (organization charts, classification hierarchies, composition models), association (semantic networks, entity relationship diagrams), cause and effect (directed graphs, fishbone diagrams, fault tree analysis diagrams), and reasoning (argument diagrams, Euler diagrams, Venn diagrams).

is essential in logic and mathematics, diagram-based reasoning, no matter how useful [Nelsen, 1993] or elegant [Polster, 2004], is not, for it is not *bona fide* reasoning. Thus, for example, Tennant once suggested a diagram is only an heuristic to prompt certain trains of inference [Tennant, 1986]; Dieudonné urged a strict adherence to axiomatic methods with no appeal to geometric intuition, at least in formal proofs [Dieudonné, 2008]; Lagrange remarked in the *Preface to the First Edition* of his *Mécanique Analytique* that no figures were to be found in his work [Lagrange, 1997]; and even Leibniz shared a similar opinion at some point (emphasis is ours):

> *La force de la démonstration est indepéndante de la figure tracée, qui n'est que pour faciliter l'intelligence de ce qu'on veut dire et fixer l'attention;* ce sont les propositions universelles, c'est-à-dire les définitions, les axiomes et les théorèmes déjà démontrés qui font le raisonnement et le soutiendraient quand la figure n'y serait pas. [Leibniz, 1966, p. 309]

This bias against diagram-based reasoning is based upon the assumption that diagrams naturally lead to fallacies, mistakes, and are not susceptible of generalization: in short, that diagrams are irrational somehow. Nevertheless, we can backtrack an argument against this assumption in Newton's Preface to the First Edition of *Principia* [Newton, 1979] by reducing proof-based reasoning to mechanical reasoning (emphasis is ours):

> But as artificers do not work with perfect accuracy, it comes to pass that mechanics is so distinguished from geometry that what is perfectly accurate is called geometrical; what is less so, is called mechanical. *However, the errors are not in the art, but in the artificers*. He that works with less accuracy is an imperfect mechanic; and if any could work with perfect accuracy, he would be the most perfect mechanic of all, for the description of right lines and circles, upon which geometry is founded, belongs to mechanics. [p. 11]

In similar lines, Allwein, Barwise, and Etchemendy [1996] and Shin [1994] have developed a successful research programme around heterogeneous and diagrammatic reasoning that has promoted different studies and model theoretic schemes that help us represent and better understand diagrammatic reasoning in logical terms, thus allowing a well defined notion of diagrammatic logical system and diagrammatic inference.

Indeed, if reasoning is a process that produces new information given previous data and information can be represented diagrammatically, it is not uncomfortable to suggest that diagrammatic inference is the unit of measure of diagrammatic reasoning: diagrammatic inference would be (in)correct depending on the compliance or violation of certain norms. Traditionally, the understanding of these norms has depended on structural and sentential approaches (semantical [Tarski, 1956a], syntactical [Carnap, 1937], and abstract [Tarski, 1956b]), but after this brief exposition a question emerges naturally: is it possible to define diagrammatic inference with structural but diagrammatic approaches?

Let us denote the relation of diagrammatic logical consequence or diagrammatic inference by $\mapsto$; this relation would define our intuitions around the informal notions of *visual inference* or *visual argument* and would follow, *ex hipothesi*, classical structural norms (reflexivity, monotonicity, and cut) and the operator $\mapsto$ would follow Shimojima's definetion of a free ride as a process in which some reasoner gains information without following any step specifically designed to gain it, i.e., as a process that allows us to reach automatically (and sometimes inadvertently) a diagrammatic conclusion from a diagrammatic representation of the premises [Shimojima, 1996, p. 32].

Using Shimojima's approach we could say that *reflexivity* establishes that a if a diagram is part of a diagrammatic configuration, then that diagram is a visual consequence of such configuration because there is a free ride from the diagrammatic configuration to a particular diagram; *monotonicity* would say that if a diagram is a free ride from a diagrammatic configuration, and a new diagram is added to such configuration, then the initial diagram is still a free ride from the original configuration.[5] Finally, *cut* would establish that if a diagram is a free ride from a diagram-

[5] In other place we have argued that monotonicity is not a *bona fide* property of diagrammatic inference, but for the purposes of this paper the assumption of monotonicity will suffice.

matic configuration and the addition of a new diagram produces a new free ride, there is a free ride from the original diagrammatic configuration to the new diagram.

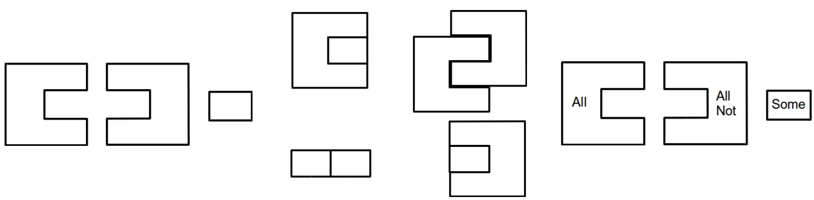
4. Słupecki's rule for diagrammatic reasoning

After this lengthy introduction, we would like show how can SR be applied to a diagrammatic interpretation of syllogistic. In order to do this, we briefly present a diagrammatic system called $L_\square$ and then we propose how can we apply SR to it.

4.1. $L_\square$

Let us suppose that syllogistic can be represented by jigsaw puzzles, that is, by tiling arrays composed by a finite set of tessellating pieces that require assembly by way of the interlocking of tiles; then, just as jigsaw puzzles require the *interlocking* of tiles, syllogisms would require the *linking* of terms.

$L_\square$ is a diagrammatic system that exploits the previous analogy by using a square-tiling tessellation—hence its name—in order to provide representation and a decision method for syllogistic. In this section we summarily introduce $L_\square$ by detailing its vocabulary, its syntax, and semantics (Figure 3).

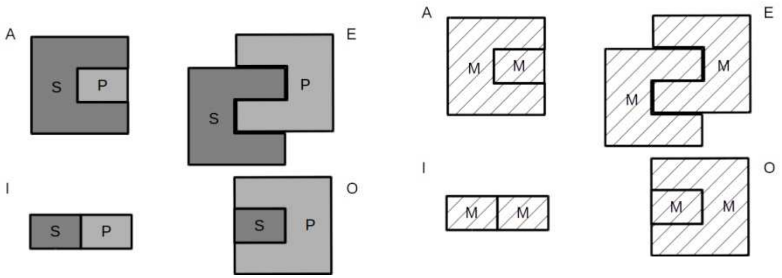


a. Vocabulary **b.** Syntax **c.** Semantics

Fig. 3. Elements of $L_\square$

The vocabulary is defined by two elementary diagrams (i.e., pieces or tiles), *sockets* and *knobs* (Figure 3a). Syntax is given by two rules: *i*) given two elementary diagrams, the combinations of Figure 3b are well formed diagrams (wfd); and *ii*) a stack of wfds is also a wfd. Semantics is given by the interpretation in Figure 3c.

With these components we can represent categorical propositions using the sockets and knobs with an implicit representation of the quantity associated to each term. For sake of brevity we label each tile with an affirmative subject or predicate term-schema, S or P. For sake of visualization, we color the terms (Figure 4a).



a. Square of opposition **b.** One term combinations

Fig. 4. General schemes of $L_{\square}$

Figure 4a also represents a square of opposition where the rules for *contradiction* between A (E) and O (I) are preserved; while the rules for *contraries*, *subalterns*, and *subcontraries* do not work, thus defining a system that behaves under a modern interpretation of syllogistic rather than Aristotelian interpretation. However, despite this shortcoming, the rules of *conversion*, *contraposition*, and *obversion* are all preserved by the mechanical operations of rotating diagrams or switching tiles.

Promptly, in order to represent categorical syllogisms and decide whether these are (in)valid, $L_{\square}$ provides a decision procedure. Suppose we build categorical propositions using a single term-schema, say M (Figure 4b). We can observe that, from these propositions, only proposition A, "All M is M," is a tautology. Using this tautology we suggest a decision procedure for $L_{\square}$ that takes any syllogism σ as an input and decides whether the given syllogism is (in)valid by verifying a single rule: if the interlocking of its middle terms produces a proposition A, the syllogism produces a free ride (i.e., it is valid); otherwise, it produces an overdetermined alternative (i.e., it is invalid) (Table 2).

Table 2. Decision algorithm for $L_{\square}$

$\mathfrak{A}(\sigma)$

Input: syllogism σ
If *interlock*(σ's middle terms)==A
    Prem(σ) $\mapsto$ *Conc*(σ)
else
    Prem(σ) ⇥ *Conc*(σ)
endIf

Using the previous decision procedure we can prove the valid syllogisms depicted in Table 3.

Table 3. Valid syllogisms

Figure 1	Figure 2	Figure 3	Figure 4
Barbara `CKUmpUsmUsp`	*Cesare* `CKNIpmUsmNIsp`	*Disamis* `CKImpUmsIsp`	*Calemes* `CKUpmNImsNIsp`
Celarent `CKNUmpUsmNIsp`	*Camestres* `CKUpmNIsmNIsp`	*Datisi* `CKUmpImsIsp`	*Dimaris* `CKIpmUmsIsp`
Darii `CKUmpIsmIsp`	*Festino* `CKNIpmUsmNUsp`	*Bocardo* `CKNUmpUmsNUsp`	*Fresison* `CKNIpmImsNUsp`
Ferio `CKNImpUsmNUsp`	*Baroco* `CKUpmNUsmNUsp`	*Ferison* `CKNImpUmsNUsp`	–

For sake of exposition, we show proof of the valid syllogisms from the Fig. 1: start by representing each premise with a well formed diagram and stack up such diagrams. Then apply algorithm $\mathfrak{A}$ to check if the middle term tiles interlock each other forming a proposition A (a step denoted by the arrows in Figure 5). Since it does so in each case, the inferences are free rides (i.e., valid), thus allowing the tiles S and P interlock in the third diagram (i.e., the conclusion).

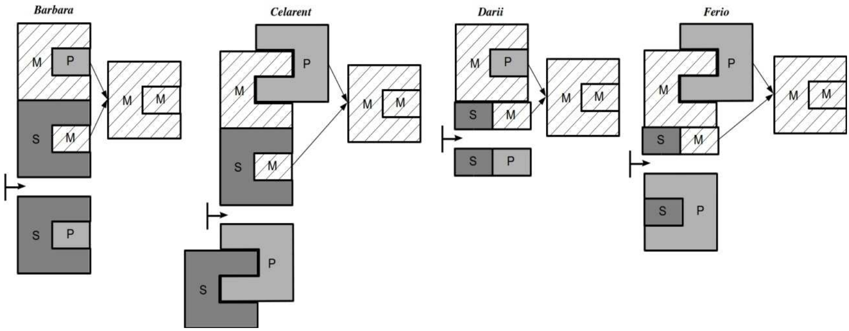


Fig. 5. Validity of the syllogisms from the Fig. 1 in $L_\square$ using $\mathfrak{A}$

Defined like so, $L_\square$ verifies the following statements:

Lemma 1 (Aristotle's Lemma) Every valid syllogism is reducible to a valid syllogism from figure 1.

Lemma 2 (Soundness w.r.t. $\mathfrak{A}$) If $\mathfrak{A}(\sigma)$=valid, then σ is valid.

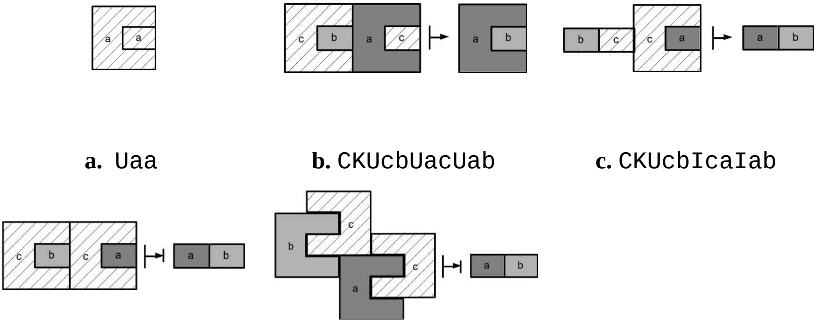
Lemma 3 (Completeness w.r.t. $\mathfrak{A}$) If σ is valid, then $\mathfrak{A}(\sigma)$=valid.

Corollary 1 (Decidability) $L_\square$ is decidable.

These results indicate that $L_\square$ (*jigsaw puzzle style*) allows us to obtain the right inferences (*soundness*) and only the right inferences (*completeness*) mechanically (*decidability*). We believe this last feature may be approached by an application of SR.

4.2. An application of Słupecki's rule

We think SR can be applied to $L_{\square}$ in three steps. First we start by showing the representation of the axioms of assertion and rejection in $L_{\square}$. We can observe that for a modern interpretation of syllogistic three axioms of assertion are correct (Figure 6a is a tautology, and Figures 6b and 6c are free rides) while the two axioms of rejection are incorrect (Figures 6d and 6e are overdetermined alternatives) by swift applications of $\mathfrak{A}$ (Figure 6).



a. `Uaa` **b.** `CKUcbUacUab` **c.** `CKUcbIcaIab`

d. CKUbcUacIab **e.** CKNIbcNIacIab

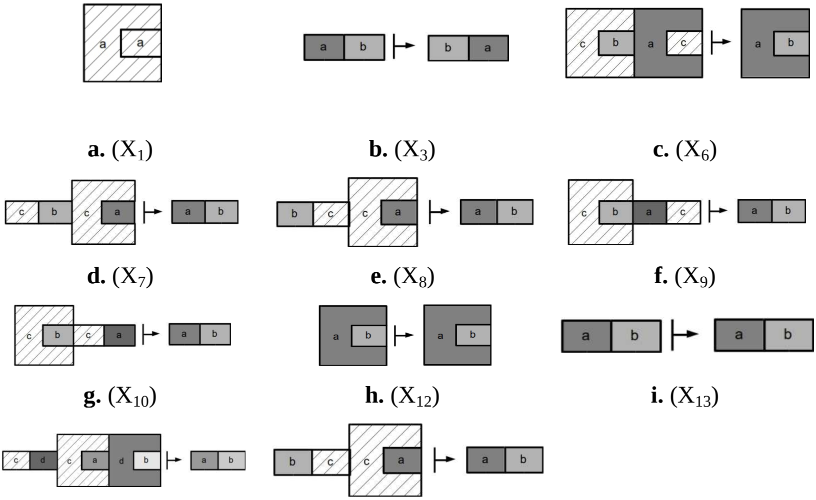
Fig. 6. Axioms of assertion and rejection in $L_{\square}$

Then, as a second step, we check that:

Lemma 4. The following meaningful expressions of syllogistic are tautologies or free rides in $L_{\square}$ (Cf. *On Aristotelian Syllogistic*, §4 Lemma X): (X_1) `Uaa`; (X_3) `CIabIba`; (X_6) `CKUcbUacUab`; (X_7) `CKIcbUcaIab`; (X_8) `CKIbcUcaIab`; (X_9) `CKUcbIacIab`; (X_{10}) `CKUcbIcaIab`; (X_{12}) `CUabUab`; (X_{13}) `CIabIab`; (X_{14}) `CKIcdKUcaUdbIab`; (X_{15}) `CKIdcKUcaUdbIab`.

Proof. We suggest proof for this statement in Figure 7 by showing that (X_1) is a tautology (Figure 7a); (X_{12}) and (X_{13}) are free rides by reflexivity

(Figures 7h and 7i); (X_3) is a free ride by conversion (Figure 7b); and the rest are free rides by applications of $\mathfrak{A}$.



a. (X_1) **b.** (X_3) **c.** (X_6)

d. (X_7) **e.** (X_8) **f.** (X_9)

g. (X_{10}) **h.** (X_{12}) **i.** (X_{13})

j. (X_{14}) **k.** (X_{15})

Fig. 7. Lemma 4

Thus, in $L_{\Box}$ the axioms of assertion are axioms or free rides, and the axioms of rejection are overdetermined alternatives. Given these two steps, we can give the third one and directly apply SR. Following Łukasiewicz, consider the rejected expressions

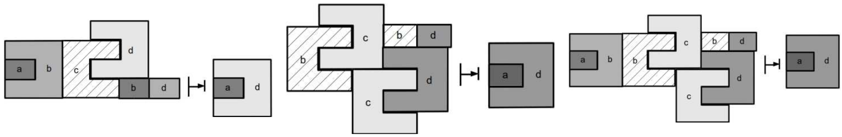
(A') *`CNAabCNIcdCIbdNAad` and (B') *`CNIbcCNIcdCIbdNAad`;

from them we get, by SR, that

(C') *`CNAabCNIbcCNIcdCIbdNAad`

must be rejected too.

Now, expressions (A') and (B') have their respective diagrammatic representations in $L_{\square}$ and, by applying $\mathfrak{A}$(A') (Figures 8a) and $\mathfrak{A}$(B') (Figures 8b) we can see, literally, that they are overdetermined alternatives.



a. $\mathfrak{A}$(A') **b.** $\mathfrak{A}$(B') **c.** (C')

Fig. 8. Overdetermined alternatives and application of SR

So, the application of $\mathfrak{A}$ disproves (A') and (B'). Hence, (C') must be rejected too, by SR (Figure 8c). And since this strategy may be applied to any case, due to the soundness and completeness of $L_{\square}$, SR holds on $L_{\square}$. Therefore, it must be that all free rides of $L_{\square}$ and no other meaningful expressions of $L_{\square}$ are valid inferences.

5. Conclusions

This application of Słupecki's rule shows that it supports diagrammatic representations and that, hence, it is not strange to suggest that diagrammatic systems do not constitute irrational approaches to logic in general, and syllogistic in particular, because they satisfy legitimate properties of decidability, which is a thesis that undercuts the claim that diagrams naturally lead to fallacies, mistakes, and are not susceptible of generalization, for such systems provide logical procedures different from but equivalent to traditional sentential approaches.

This adds some modest reinforcement to the projects of Allwein, Barwise, and Etchemendy [1996] and Shin [1994] and shows that Slupecki's advances to syllogistic are quite fertile and that, although the original Warsaw school of logic ceased to exist, Polish logic may live forever.

Acknowledgements

We would like to thank the anonymous reviewers for their precise corrections and useful comments. Financial support given by UPAEP Grant 30108-1008.

References

Allwein G., Barwise, J., & Etchemendy J., (1996), *Logical Reasoning with Diagrams*, New York, Oxford University Press.

Bök A.F., (1766), *Sammlung der Schriften, welche den logischen Calcul Herrn Ploucquets betreffen*, Frankfurt.

Carnap R. (1937), *The Logical Syntax of Language*, Trans. A. Smeaton, London, Kegan Paul, Trench, Trubner & Co.

Carroll L., (1887), *The Game of Logic* (2nd ed.), London, Macmillan and Co.

Descartes R., (1982), *Principles of Philosophy*, Translated by V.R. Miller, R.P. Miller, Dordrecht-Boston-London, Kluwer.

Dieudonné J., (2008), *Foundations of Modern Analysis*, Read Books.

Gardner, M., (1958), *Logic Machines and Diagrams*, New York-Toronto-London, McGraw-Hill.

Hamilton W., (1866), *Lectures on Metaphysics and Logic*, Boston: Gould and Lincoln.

Lagrange, J.L., (1997), *Analytical Mechanics*, Translated and edited by A. Boissonnade, V.N. Vagliente, Dordrecht, Springer Science+Business Media.

Lambert J. H., (1764), *Neues Organon*, Leipzig.

Larkin J. H. & Simon, H. A., (1987), "Why a Diagram is (Sometimes) Worth Ten Thousand Words", *Cognitive Science* 11(1), pp. 65-100.

Leibniz G., (1966), *Nouveaux essais sur l'entendement humain*, Paris, Garnier-Flammarion.

Leibniz G., (1903), *Opuscules et fragments inédits de Leibniz. Extraits des manuscrits de la Bibliothèque royale de Hanovre par Louis Couturat*, Paris, Félix Alcan.

Llull R., (1501), *Ars magna*, Impressum per Petru[m] Posa.

Londey D. & Johanson, C., (1987), *The Logic of Apuleius*, Philosophia Antiqua No. 47, Leiden, Brill.

Łukasiewicz J., (1951), *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, Clarendon, Oxford.

Łukasiewicz J., (1970), *Selected Works*, Borkowski L. [ed.], Amsterdam, North-Holland.

Nakatsu R., (2009), *Reasoning with Diagrams Decision-Making and Problem-Solving with Diagrams*, Hoboken, John Wiley & Sons.

Nelsen R., (1993), *Proofs Without Words: Exercises in Visual Thinking*, The Mathematical Association of America.

Newton I., (1979), *Mathematical Principles of Natural Philosophy*, Translated by A. Motte, University of California Press.

Polster B., (2004), *Q.E.D.: Beauty in Mathematical Proof*, New York: Wooden Books.

Tarski A., (1956a), "On the Concept of Logical Consequence", [in:] *Logic, Semantics, Metamathematics. Papers from 1923 to 1938*, Oxford, Clarendon.

Tarski A., (1956b), "On Some Fundamental Concepts of Metamathematics", [in:] *Logic, Semantics, Metamathematics. Papers from 1923 to 1938*, Oxford, Clarendon.

Tennant N., (1986), "The Withering Away of Formal Semantics", *Mind & Language* 1(4), pp. 302–318.

Shimojima A., (1996), "Operational Constraints in Diagrammatic Reasoning", [in:] G.Allwein, J. Barwise, & J. Etchemendy [eds.], *Logical Reasoning with Diagrams*, New York, Oxford University Press.

Shin S., (1994), *The Logical Status of Diagrams*, Cambridge, Cambridge University Press.

Słupecki J. Z., (1951), "On Aristotelian Syllogistic", *Studia Philosophica* 4, pp. 275-300.

Stevin S., (1586), *De Beghinselen der Weeghconst*, Leyden.

Venn J., (1880), "On the Diagrammatic and Mechanical Representation of Propositions and Reasonings", *London, Edinburgh and Dublin Philosophical Magazine and Journal of Science* (5 Ser.) 10, pp. 1-18.

Woleński J. & Zygmunt, J., (1989), "Jerzy Słupecki (1904-1987): Life and Work", *Studia Logica* 48, pp. 401-411.

José Martín Castro-Manzano,
Universidad Popular Autónoma del Estado de Puebla (UPAEP), Mexico,
josemartin.castro@upaep.mx