STUDIA METODOLOGICZNE
NR 35 • 2015, 151-165
DOI: 10.14746/sm.2015.35.11

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Kazimierz Ajdukiewicz on Interrogative Sentences and the Rationality of Assumptions

ABSTRACT. We show some ideas from the Polish philosopher Kazimierz Ajdukiewicz about interrogative sentences and some classifications of answers. We resort to First Order and Doxastic Logic in order to express some of his results and we also recourse to normal and non normal squares and hexagons of opposition to express his findings. Finally we say some words about a relationship found in the medieval octagons of opposition which is not found in the traditional square; it is found also in our doxastic hexagon of presuppositions.

KEY WORDS: Ajdukiewicz, interrogative sentences, presuppositions, doxastic, oppositions

1. Introduction

The purpose of this study is to show some ideas on interrogative sentences developed by the Polish philosopher Kazimierz Ajdukiewicz (1890–1963) [Ajdukiewicz, 1978, pp. 155–164; Ajdukiewicz, 1974, pp. 85–94]. Parts 2–5 present informally his ideas on questions, almost in his own words; parts 6–10 develop a formal analysis which involves first order logic plus doxastic logic and which organize his ideas using squares and hexagons of opposition. We will use standard and non-standard hexagons of opposition to cope with Ajdukiewicz's ideas about the complexity of interrogative sentences and the assumptions made when asking certain kinds of questions.

2. The structure of questions and sentential functions

Indicative mood sentences have truth value, but interrogative sentences do not. Let us start by saying that the answer to a question is an indicative mood sentence, or rather, a set of indicative mood sentences, since a question may have several answers; these can be classified into proper and improper ones. We will return to this.

In each question there may be a sentence fragment or a complete sentence; they may also have an interrogative particle, being either a pronoun or an adverb, plus the question marks. For instance

Is the Earth round?

The parts of the previous interrogative sentence may be arranged in such a way that we obtain a complete indicative mood sentence, "The Earth is round". We have a part of an indicative mood sentence in the following question

Who discovered America?

If we take away the interrogative pronoun, we obtain a part of the sentence. I show this by removing the pronoun and leaving a blank,

"____ discovered America".

Which sentences could count as an answer to who discovered America?

Before answering this I would like to show what I think is the basic idea of Ajdukiewicz´s approach: when we remove the interrogative pronoun in the former sentences we obtain a blank. This is exactly what is called a *sentential function* in Symbolic Logic, a grammatical structure which is not a sentence on its own and contains a blank or an x variable which when properly filled or substituted becomes a sentence. The sentential function, the structure sentence containing a blank, becomes a sentence when filling the blank with a proper name or with a single expression; it could be a definite description also. We also obtain a sentence when filling the blank with universal or particular quantifiers, but we should not resort to quantifiers since they are already taken into account as a sort of presupposition, as we shall see later. Let us go back to our question.

In the sentential function

“____ discovered America”.

How could we fill it to produce an answer?

The function allows a set of answers, and this set contains individual names that can substitute the variable producing a proper answer to the question. All of these are proper answers forming a set of answers:

“Magellan discovered America”
“Julius discovered America”
“Columbus discovered America”
“Napoleon discovered America”

since all of them satisfy the function

“_x_discovered America”.

There are false sentences among them and one true sentence, but each one follows the same pattern.

So there is a question and we do not know the answer, but, Ajdukiewicz says, we do know the structure of the answer. We also know that the question looks for proper names as substitutional instances of the sentential function

Who discovered America? “_x_discovered America”

The answer will be a substitutional instance of the sentential function.

The structure of the answer is established by the fragment of the sentence that contains the question. The function is determined by the fragment and the interrogative particle indicates where the variable x is to be placed.

3. Questions asking for direct complement and adverbs

Let´s take some other questions:

Who killed Caesar? Answer: _x_ killed Caesar

Whom did Brutus kill? Answer: Brutus killed _x_

The second question asks for the direct object of the sentence.

The sentential function resulting from the question is called by Ajdukiewicz *datum questions*, "the given of the question", the information given by the question; it gives us the structure of the answer. The set of values specified by the interrogative pronoun or by an adverb or some other specification is called the *range of the unknown*.

Let us go to another kind of question, this time forming a complete sentence

How do lamps shine?

We have the interrogative particle *how* and the sentence (*the) lamps shine*.

We have here an interrogative particle, *How*, and a complete sentence, (*The) lamps shine*.

The range of the variable is constituted by a set of adverbs, for instance, "adequately", "poorly", "brightly". Thus, the *datum quaestionis* also works when we have a complete sentence inside the question.

There are decision questions, asking for a yes/no, all/none answers, and complementary questions, like the former examples.

The questions:

Does the sun shine?
Is the whale a fish?

result in these mutually contradictory answers:

The sun does shine/the sun does not shine
The whale is a fish/the whale is not a fish

Ajdukiewicz's approach could be generalized to other grammatical structures.

4. Pragmatic issues

It is advisable, says Ajdukiewicz, to indicate with no ambiguity the *datum quaestionis* and the range of the unknown. This reminds one of Grice and his Cooperative Principle. We could establish some requirements for posing questions and the things presupposed by doing so, *mutatis mutandis* in an analogous way as Grice´s maxims for his Cooperative Principle (see Grice 1975). Ajdukiewicz says something about the conditions for clarity, such as stating clearly the range of the unknown and the *status quaestionis* when posing a question: "When these are not indicated, then the person to whom the question is addressed does not know what he is asked about." [Ajdukiewicz, 1974, p. 87]

Some assumptions are made when asking a question. If one asks seriously "Who killed Caesar?" he or she presupposes that somebody killed Caesar (a positive assumption) and somebody did not kill Caesar (a negative assumption). Someone but not all. Notice that believing that everyone did it cancels the question. A question shows the belief of the questioner precisely through the positive and negative assumptions; in this sense interrogative sentences may be used to communicate information. Let us give an example.

Somebody asks me: When did John get married?

Even though I know John, I knew nothing about his marriage, so the question itself gives me some information. Ajdukiewicz calls *suggestive questions* those questions made to communicate information the listener does not know, especially at the level of positive and negative assumptions. At this point, however, the confidence of the listener in the person who formulated the suggestive question is basic. Information is also provided by words, gestures, or intonation. Suggestive questions can be malicious when they suggest a false answer.

Ajdukiewicz is aware of the complexity of questions and answers according to different situations. A teacher may ask questions during an examination, and he already knows the answers, so in a sense they are not real questions; but this is quite different from the student´s point of view. The teacher may pose a real question (i.e. asking for something that he or she ignores) when asking “Do you know the answer to that question?”.

One person who has lost her umbrella may ask “Where is my umbrella?” and someone else may be around and listen to the question. The psychological importance is quite different since the person who asks it is in “a state of tension directed towards acquiring a suitable item of information”; while the second one is not in that state of tension [Ajdukiewicz, 1978, p. 162). By the way, this state of tension is described as thirst:

> The thought expressed by a person by means of an interrogative sentence is usually that of a mental tension, similar to thirst; it is a state in which that person strives to develop a conviction that may be expressed by a proper answer to that interrogative sentence [Ajdukiewicz, 1974, p. 91].

Just as thirst is something to satisfy doubt, also mental tension which implies a question is something to be overcome. In Peirce’s words:

> The irritation of doubt is the only immediate motive for the struggle to attain belief… With the doubt, therefore, the struggle begins, and with the cessation of doubt it ends [Peirce, 1955, p. 10].

5. Some classification of answers

Answers may be proper and improper. They are proper when they are obtained from the *datum quaestionis* substituting the variable by some value of the range of the unknown. If this does not happen, they are improper. Nevertheless, it may satisfy some questioner´s expectations, for instance, the indirect answers:

> Question: Is the whale a fish?
> Answer: The whale is a mammal.

The question is a decision question: yes/no, but the answer implies the proper answer: the whale is not a fish.

We have also partial answers

> Question: Who discovered America?
> Answer: An Italian discovered America.

The answer does not imply a proper answer, but it does exclude some proper answers (for instance, “Magellan discovered America”, since he is not Italian). The following is also a partial answer:

> Question, made by a professor: “Who did it?”
> A student reply: “I haven´t.”

There are answers that refute the positive assumption with an answer that contradicts it or that involve a sentence that contradicts it:

> Question: Who was Copernicus´ son?
> Answer: Copernicus had no son.

which refutes the assumption: Copernicus had a son.

Thus, there are questions badly formulated, ill posed, like those with some false assumptions. In these cases there can be no answers, not even partial ones. What we are left with is to refute the assumptions.

The exhaustive answers are real answers and involve each proper answer.

6. The positive and negative assumptions of a question

When we ask *who*? we make an assumption, which is this: at least one proper answer is true.

> Who discovered America?
> Magellan did… or Julius… or Columbus… or Napoleon… etc.

The positive assumption is: someone discovered America. Of course, we may also make a negative assumption, someone did not discover America. We make positive and negative assumptions because we suppose the one who asks it does it seriously, he believes that some but not all proper answers are true.

The assumptions may be arranged in a traditional Square of Opposition, according to the usual sentences: universal affirmative, universal negative, particular affirmative and particular negative. The positive and negative assumptions correspond to the subcontrary sentences from the square, the affirmative and negative particular ones.

These assumptions allow us to say that their contradictories, that is, the universal sentences, are false, since not all discovered America; it is also false that no one did. The particular sentences are both true since someone did it (Columbus) and someone did not (Magellan, for instance). In this square of assumptions universal sentences are false and particular sentences are true (at least they are believed so).

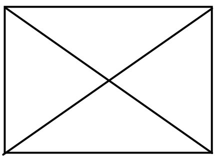


Fig. 1.

7. Two hexagons of opposition

There are at least two ways to expand the square of opposition into a hexagon. The first way consists in the addition of two singular and contradictory sentences *inside* the square. A singular sentence, which is implied by the universal affirmative quantifier (“Julius discovered America” for instance.), is placed between the A and I corners. A singular negative sentence implied by the negative universal quantifier (“Julius did not discover America” for instance) is placed between the E and O corners; I shall call them *a* and *e* respectively. These singular sentences imply the particular sentences, so a hexagon is formed. Implications (subalternations) go from up to down.

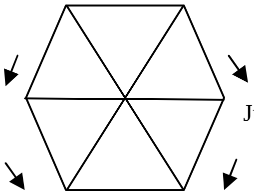


Fig. 2.

The second way consists in the addition of sentential connectives that join a pair of sentences *outside*, at the bottom and above the square. Universal, i.e. contrary sentences, are joined by a disjunction and placed outside and above the square (“Everyone discovered America OR no one discovered America”); particular, i.e. subcontrary sentences, are joined by a conjunction and are placed at the bottom of the square (“Someone discovered America AND Someone did not discover America”); these sentences are mutually contradictory. Using the usual letters A, E, I and O and Blanche´s Y and U letters for the new contradictory corners, we obtain this hexagon, where the implications go bottom–up:

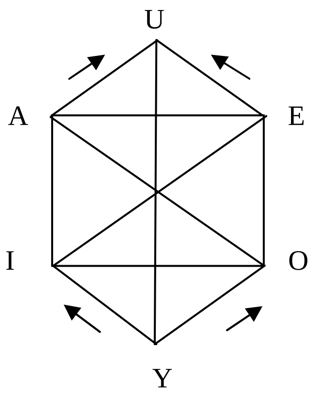


Fig. 3.

The first hexagon was already suggested by William of Sherwood in the 13th Century [Sherwood, 1995, chap.1]. Sherwood adds singular sentences between universal and particular sentences and establishes their oppositions with the quantified extremes [see Khomskii, 2012, p. 49]. The second octagon has been suggested by R. Blanche and it has had a successful history [see Béziau, 2003, p. 220]. We can express the positive and negative assumptions precisely by the Y sentence, and its contradictory corresponds to the U sentence above. Y is true and U is false.

8. Sherwood's hexagon and the *datum quaestionis*

We could incorporate the *datum quaestionis* and its negative counterpart into a hexagon similar to Sherwood´s, placing them at the half way point between universal and particular sentences. Using the standard Logic symbolism, where *a*: America, Dx*a*: x discovered *a*, ∀x: universal quantifier, ∃x: particular quantifier and ~ is for negation, and thus we obtain:

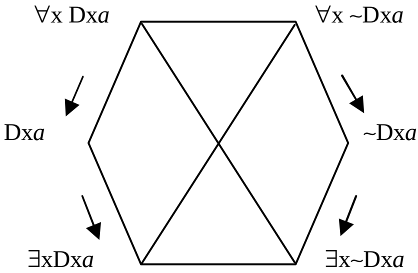


Fig. 4.

We should note this: Sherwood´s hexagon consists of six sentences, four are quantified and two are intermediate singular sentences. In the hexagon above we have four quantified sentences and two sentential functions, which are neither true nor false. They are true or false when substituting the variables for constants (a proper name in this case) or when they are quantified; both possibilities are excluded here. Sentential functions are neither contradictory nor may oppose other sentences though they have the syntactic structure of a contradiction (Dx*a* and ~Dx*a*). In this sense the hexagon with *data quaestionis* is a non-normal hexagon, since a part of its elements maintains no opposition to the others. By the way, the negative version of the *datum quaestionis* comes from the "negative" question "Who did *not* discover America?" We can say that a Sherwood type hexagon captures the two sides of a question.

The Hexagons help us to situate the *data quaestionis* and the positive and negative assumptions. Particular sentences may be expressed as disjunctions of singular sentences, as we have seen before: "Magallan discovered America or Julius discovered America or Columbus discovered America". Universal sentences may be expressed as conjunctions of singular sentences: "Magellan discovered America and Julius discovered America and Columbus discovered America". This is why the hexagon can be expressed with no quantifiers, provided we have a finite number of individuals. This is a normal hexagon, seeing that the internal sentences are the only difference between figures 4 and 5.

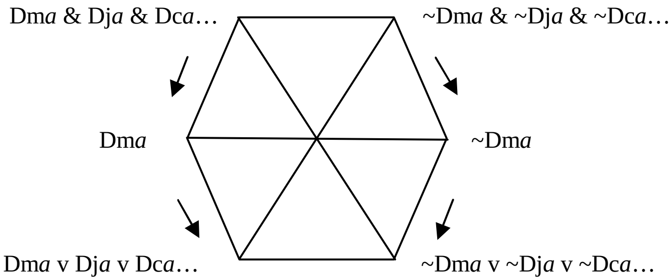


Fig. 5.

9. Doxastic hexagon of presuppositions

We should notice that positive and negative assumptions need Doxastic Logic operators, Belief B and Doxastic compatibility C, i.e. strong and weak operators. They follow this equivalence[1]

Bp S iff ~Cp~S

A person p believes a sentence S iff it is not the case that ~S is compatible with all that p believes.

The person who asks – let us call her p – who discovered America?, *believes* the Y type sentence, that is

Someone discovered America AND Not all discovered America

$Bp\exists xDx\mathit{a}$ and $Bp{\sim}\forall xDx\mathit{a}$ (i.e. $Bp\exists x{\sim}Dx\mathit{a}$)

[1] See Hintikka´s rule (C. ~B) [see Hintikka, 1962, pp. 69, 125].

Their contradictories (the U sentence) are: $\sim Bp\exists xDxa$ and $\sim Bp\exists x\sim Dxa$, which are equivalent to

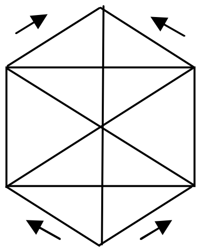
$$Cp\forall x\sim Dxa \quad \text{and} \quad Cp\forall xDxa$$

We thus obtain

U

$Cp\forall xDxa \vee Cp\forall x\sim Dxa$

$Cp\forall xDxa$ $Cp\forall x\sim Dxa$



$\exists xDxa$ $Bp\exists x\sim Dxa$

$Bp\exists xDxa \;\&\; Bp\exists x\sim Dxa$

Y

Fig. 6.

But p does *not believe* that:

Everyone discovered America: $\sim Bp\forall xDxa$, equivalent to $Cp\exists x\sim Dxa$
No one discovered America: $\sim Bp\forall x\sim Dxa$, equivalent to $Cp\exists xDxa$

Notice that what the person p believes are particular sentences and what she does not believe are universal sentences. What the person does not believe is consistent with the fact that she believes, as we can see through the equivalencies just mentioned, which read: It is consistent with all that p believes that someone discovered America and it is consistent with all that p believes that someone did not discover America.

10. A new relationship inside the doxastic square

The doxastic hexagon is a non-normal hexagon since there is something unusual in it. In effect, the usual subalternations do not hold up here. Let us isolate the square inside the hexagon to see what is happening.

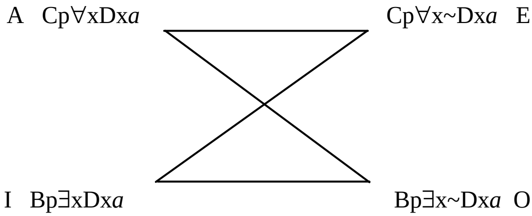


Fig. 7.

Let us describe the sentences of the square as composed of a doxastic operator (either belief or consistency), a quantifier (either universal or particular) and the *datum quaestionis* (either affirmative or negative). I shall call "strong operators" to B and ∀x, and "weak operators" to C and ∃x. We may as well consider the strong as universal and the weak as particular. We know that the strong imply the weak and not *vice versa*. We may omit the *data quaestionis* since there is no need of them in our following analysis.

Sentence A is universal because of the quantifier, but particular because of the doxastic operator; the same holds for E. Sentence I is universal because of the doxastic operator, but particular by the quantifier; and the same holds for the O sentence. The A and E sentences are actually weak, since the C operator governs the whole sentence in both cases. Sentences I and O are strong for they are B governed. We have an upside down situation here.

A: Weak Strong   E: Weak Strong
↑ ↓   ↑ ↓
I: Strong Weak   O: Strong Weak

The arrows show what could be implied inside each sentence, from strong to weak, or from universal to particular. On the whole there is no implication; there is no other opposition either. They may be regarded as logically independent sentences and they are what the medieval logicians called *disparatae*, which are to be found in the medieval octagons of opposition (for a description of and rules on these sentences see Campos 2014: p. 364).

This suggests, I guess but I'm not sure, that questions and their assumptions do not behave, logically speaking, in the same way as indicative sentences do. Anyway, Ajdukiewicz's ideas on interrogative sentences are very suggestive and deserve our full attention.

Bibliography

Ajdukiewicz K., (1974), "Questions and Interrogative Sentences", [in:] K. Ajdukiewicz, *Pragmatic Logic*, [ed.] O. Wojtasiewicz, Warsaw, PWN, Dordrecht–Boston, D. Reidel Publishing Company, pp. 85–94.

Ajdukiewicz K., (1978), "Interrogative Sentences", [in:] K. Ajdukiewicz, *The Scientific World-Perspectivs and other Essays, 1931–1963*, [ed.] J. Giedymin, Dordrecht-Boston, D. Reidel Publishing Company, pp. 155–164.

Béziau J.-Y., (2003), "New Light on the Square of Opposition and its Nameless Corner", *Logical Investigacions* 10, pp. 218–232.

Campos Benítez J., (2014), "The Medieval Octagon for Sentences with Quantified Predicates", *History and Philosophy of Logic*, Vol. 35, Issue 4, pp. 354–368.

Grice, Herbert P., (1975), "Logic and Conversation", [in:] *Syntax and Semantics*, P. Cole, J. Morgan [eds.], New York Academic Press, pp. 41–58.

Hintikka I., (1962), *Knowledge and Belief. An Introduction to the Logic of the Two Notions*, Ithaca, Cornell University Press.

Khomskii Y., (2012), "William of Sherwood, Singular Propositions and the Hexagon of Opposition", [in:] *The Square of Opposition. A General Framework for Cognition*, [eds.] J-Y. Béziau, G. Payette, Basel, Peter Lang International Academic Publishers, pp. 43–59.

Peirce Ch.S., (1955), "The Fixation of Belief", [in:] *Philosophical Writings of Peirce*, [ed.] J. Buchler, New York, Dover Publications, pp. 5–22.

Sherwood W. of, (1995), *Introductiones in Logicam*, [eds.] H. Brands and C. Kann, Hamburg, Meiner-Verlag.

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