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$\mathfrak{L}_{\alpha}$: A Modal Logic to Reason about Analogical Proportion

ABSTRACT. In [Prade and Richard, 2009] a restricted study of analogy was developed through the notion of analogical proportions, i.e. sequences of the form "*a is to b as c is to d*". They define three kinds of *analogical proportions*: analogy, reverse analogy, and paralogy. In [Prade and Richard, 2013] and [Prade and Richard, 2014] many kinds of analogy are defined but we highlight four: analogy, reverse analogy, paralogy, and inverse paralogy. In all of these works analogy is analyzed in a Boolean sense taking an account of analogy in a logical terms.

Our hypothesis is that if we take a restricted notion of analogy in the sense of the mentioned works, analogy could be seen as a modal operator. We proceed as follows. In the first section we present a background of the notion of analogical proportion, we take the main thesis of Henri Prade and Gilles Richard in the mentioned works. Later, in the second part of the paper we present the basic system of analogical proportions: the logic $\mathfrak{L}_{\alpha}$. We define a modal propositional language with four basic modal operators, then, we present a model based on a relational structure with two types of relations defined as two kinds of accessibility relations between states. Our technique is to interpret analogical proportions as dyadic relations between pairs of objects holding an inclusion relation. In this sense, the formulas related by the analogical modal operators are truth in states that hold some analogical proportion.

KEY WORDS: modal logic, analogical proportion, homogeneous analogy, classical propositional logic.

Introduction

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