STUDIA METODOLOGICZNE
NR 37 • 2016, 97–120
DOI: 10.14746/sm.2016.37.6

ARTURO ROMERO CONTRERAS

Analogy and Mapping: Philosophy, Mathematics and Space

ABSTRACT. In this paper I aim to show that the classic concept of "analogy" can be interpreted in mathematical terms. The vagueness of how "alike" two objects are, can be tackled by a consideration of their topological and group properties, especially symmetry and connectiviy. Two objects can be put in a relationship of mapping, and the likeness would depend on which properties are preserved through the morphism, including their local and/global character. The concept of analogy plays a key role in Aristotle and scholastic philosophy. In this philosophical tradition it is stated that some concepts are *univocal* and some are *equivocal*. Analogy is understood as a third term between pure difference and pure identity. But a problem arises however when resorting to more strict uses of analogical reasoning: it lacks of rigorosity. Not because science cannot employ analogies between realms, but because they cannot be evaluated. There are no objective degrees of likeness or at least criteria to evaluate how adequate or inadequate an analogy is. It is in the Renaissance philosophy however, where analogy gains a radically new significance, as it is linked to mathematical *structures*. It was not only proportion or metaphor, but a more general term which emerged progressively, namely, "form". Analogy was not to be settled upon vague and questionable resemblances—of qualitative nature—nor in pure quantitative terms—as in the case of proportion. Settled the ground for the discussion, some basic notions of topology and group theory are presented. The core idea of this section is the concept of map as a way of putting two different spaces in correspondence. With some mathematical elements we offer a model which depicts the double relationship of a subject to the world and to another subject, such that an ontological as well an intersubjective approach can be articulated. This model takes inspiration in polycontextural reasoning, a non-classic logic. In the next section I discuss the property of connectivity in polycontextural logic in contrast to the classical Aristotelian approach. I conclude with some phenomenological reflections to interpret the discussion carried above as a way of understanding the world and our experience in general.

KEY WORDS: analogy, topology, phenomenology, non-classical logics, intersubjectivity

1. Introduction

In this paper I claim that the philosophical concept of *analogy* a) can be interpreted in mathematical and, more specifically, geometrical terms and b) that this mathematical framework has fundamental philosophical implications. Analogy has played many roles in the history of philosophy and it has been interpreted differently in a wide range of contexts. Regarding the use of analogy in philosophy and in science is has been pointed out many times the lack of rigorous criteria to apply it. But if we understand analogy through mathematical concepts like symmetry, space, connectivity and structure, we may count with important elements to *assess* scientific and philosophical uses of it. This concerns the first aim of the article, namely, the framework in which can we interpret analogy. The second aim is to ask, whether analogy, already interpreted through mathematical categories, can be philosophical meaningful, more concretely, if it sheds new light on how we conceive of ontology. Both aims are ambitious and cannot be fulfilled in an article. I intend however to provide some important insights to link analogy and mathematics and to offer directions to develop philosophical concepts relevant to ontological questions.

The article is organized as follows: first I introduce the concept of analogy as we find it in Aristotle. Then I proceed to show how analogy starts being formalized through Renaissance's geometry. This lays the ground to understand analogy as a mapping between spaces. Once in the realm of mathematical maps, I introduce the concept of symmetry through some basic diagrams. At this point, provided the elementary mathematical concepts, I try to apply them to the classical ontological structure of subject-object (objectivity) and subject-subject (intersubjectivity). The question is how to read and expand these classical structures anew through a type of mathematical analogy. To ground this remarks in a philosophical tradition and framework I chose Husserl´s phenomenology, where analogy seems to be at the core of ontology.

2. Some generals remarks on Aristotle's concept of analogy

The concept of analogy, as it is well known, plays a key role in Aristotle and scholastic philosophy. In the former, being is structured by relationships of genus and species in a vertical tree-like structure. Analogy, however, allows a sort of horizontal linking of beings. Originally, analogy meant so much as proportion, like in the case A is to B as C is to D. Or, in its abbreviated form, as it happens in the so-called golden-ratio: A is to B as B is to AB. But there is in Plato and later in Aristotle's *Rhetoric*[1] and *Prior Analytics*[2] an "extension" from a pure quantitative to a qualitative use of analogy. Aristotle speaks of two types of analogy: *paradeigma* and *homoiotes*, both capable of being used in deductive arguments.

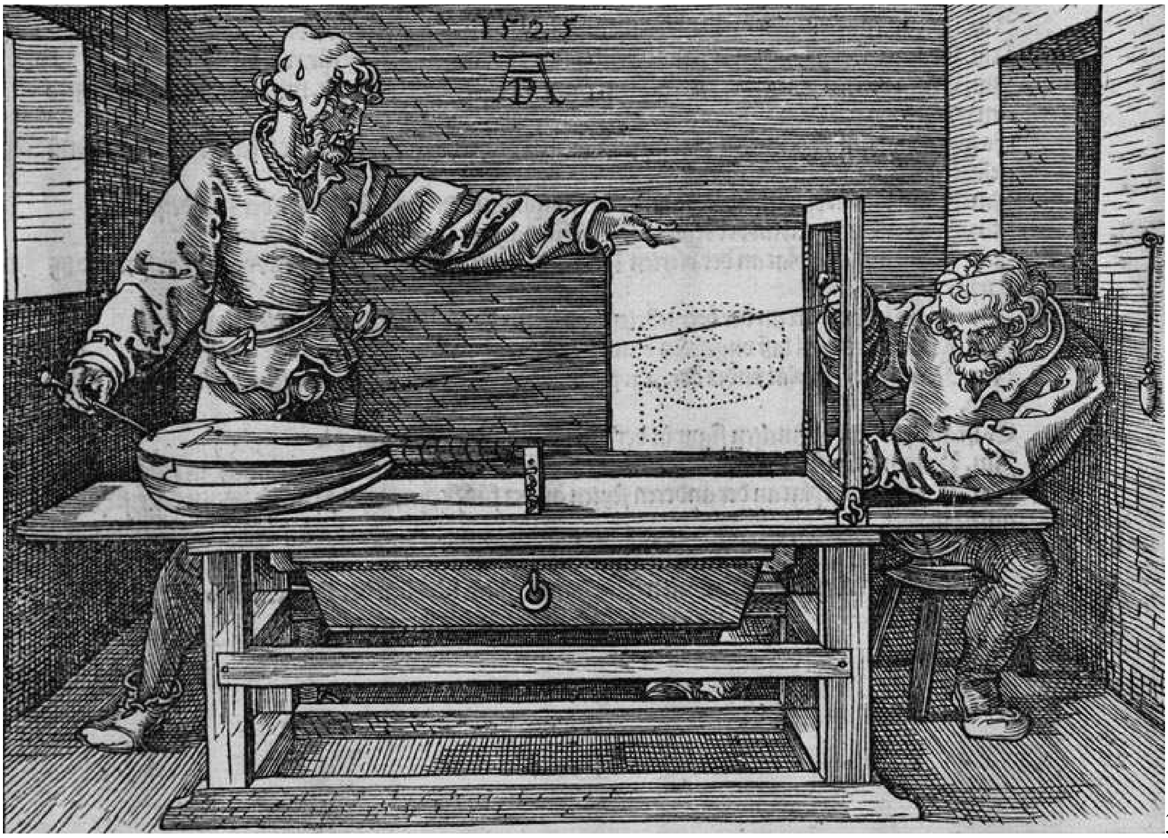
But we should not interpret analogy in a pure, linguistic way. In Aristotle, categories are necessarily both linguistic *and* ontological. In medieval thought it is stated that some concepts are *univocal* and some are *equivocal*. Univocity means that there are no *degrees of freedom* to interpret a word i.e., there is only one possible sense. Equivocity means on the contrary that words may be *polyvalent*, and that we cannot find the common ground for the resulting multiplicity. But there is a *third* term between pure difference and identity, this is analogy. As in Aristotle, analogy allows to *link beings* in a semi-proper manner. I claim that metaphysics should be understood as a "science of the common as such", as a *koinology* or *communology*, for being is nothing but the *connectivity* and *communicativity* between all beings. In this sense, we could say that classical ontology thought being as univocity. The philosophy of difference thought, in contrast, being through the other of univocity, i.e., equivocity. What we have to think is a way beyond this opposition. This is what analogy offers us.

3. Analogy and renaissance

Analogy has a rich history in philosophy, but there is a risk when we try to provide a more formal approach. It can be conceived of as a *mere* metaphor and metaphors lack of rigor. This is not because science cannot resort to analogies between realms, but because analogies and metaphors cannot be *assessed*. There are no objective degrees of likeness or at least criteria to evaluate how precise or imprecise an analogy is.

[1] [Aristotle, 1959].
[2] [Aristotle, 1989].

It is in the Renaissance philosophy, however, where analogy gains a radically new significance, as it is linked to mathematical *structures*. Indeed, there was surely an indiscriminate use of vague similarities between the farthest regions of being, especially between the macro- and the micro-world, between cosmos and man. Many superficial connections were established, so that nature showed correspondences in all scales and places. But at the same time, such resemblances, in contrast to medieval thought, were more and more expressed in terms of *mathematics*. This was not only in terms of proportion or metaphor, but a more general term which emerged progressively, namely, "form" (and pattern). Analogy was not to be settled upon vague and questionable resemblances – of a qualitative nature – nor in pure quantitative terms – as in the case of proportion.



[3]

Fig. 1.

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[3] Dürer Anweisung zur Messung mit Zirkel und Richtscheidt (1. Ausgabe) (1538). Public domain: https://commons.wikimedia.org/wiki/File:D%C3%BCrer_Stich_aus_Anweisung_2.jpg Consulted: March 20 2016.

Drawing our attention to renaissance painting, one can notice at first glance that perspective is nothing but an instrument to produce the *effect* of depth in a painting. It is basically a *trompe l'œil*. But mathematically there is something different happening. We are *projecting*, or *mapping*, our experience-world onto another space, namely, that of projective geometry, which corresponds to a non-Euclidian space.[4]

In the picture above we see a work of Albrecht Dürer, depicting the technique of projection to obtain the effect of perspective on a picture. We speak of a non-Euclidian space because the parallel postulate does not hold. All parallel lines intersect in the so-called point to infinity. The fundamental contribution of renaissance painting is, as we have said, the idea of mapping one space onto another. From this moment on, it will be clear that figures and their properties are not independent of the space in which they are inscribed. We could risk an ontological generalization saying that no presented object is independent of the space in which it appears. There is no neutral phenomenology, but a multiplicity of spaces. But since there are multiple spaces, there must be a way to *connect* them. There is always more than one space and, of course, more than one way to translate one onto the other. In this sense mapping is equivalent to translating.

We should not speak here of representation, but of mappings or morphisms. Now, what is the relationship between our lived world – a mixture between the Euclidian and non-Euclidian world – and the picture? Could we speak of analogy? Indeed, we could – but (only) in the very special sense of partial or non-perfect *mapping*. What is mapping here? It is a transformation of one figure into another – by rotation, stretching, or putting into perspective – or of one space into another – via immersion or embedding

It is in this sense that mathematics opens up a new door to deal with the ancient philosophical issue of analogy. But before we explain this further, some clarification is needed about how we understand equivocity and univocity and their relationship to analogy.

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[4] See: [Edgerton, 1975].

4. Equivocity and univocity

Analogy is a third term, an intermediate level between univocity and equivocity. But how should we conceive of univocity in the first place? Univocity implies the notion of "sameness". A concept is univocal if and only if it does not allow different interpretations, i.e., if it is absolutely determined or defined such that it does not allow different values. But this is a rather narrow concept of sameness, for it gives the idea of very rigid concepts. If we try to formulate the idea of sameness in group-theory, what we obtain is a *group* of transformations.

What does this mean? It means that an "object" may suffer different transformations without changing its structural properties. Let´s take the trivial example of an equilateral triangle. We can rotate it 120 degrees every time and we will always "see" the same triangle. We can flip it horizontally with the same result. All these transformations constitute a so-called group.

Here we see operations of rotation and reflection on the equilateral triangle[5]:

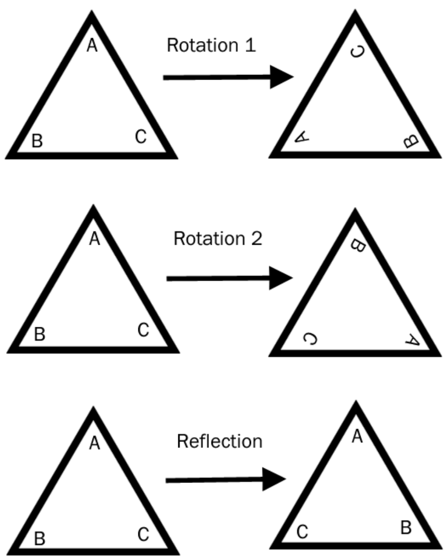

Fig. 2.

[5] All images are mine, except when indicated otherwise.

In group theory, invariants are the focus of attention. Transformations map one set X (domain) onto another set Y (codomain), but they may also be understood as mappings of a set X onto itself.

We could take, of course, other types of transformations, which seem at first sight to render different objects. Examples of transformations are translation, reflection, rotation, scaling or shear. Important in every case is that figures, or more precisely, *spaces*, even if they look very different, belong to the same group of possible *transformations*. Sameness does not seem to lead us to a *single* object, but to a group of possible variations. Now, we cannot always establish with ease if two objects are the "same". Actually, the question is, if we can *continuously transform* one into the other, and if in this transformation the defining properties of the objects are preserved.

We move now to equivocity. Equivocity means difference. It means, however, not only that a notion may possess different meanings, but that there is an *irreducibility of plurality*. Difference means, radically thought, that a variety of notions have nothing in common. Neither unity nor identity may be applied to them; they do not constitute a "category", or a "set", in a proper sense (we cannot define a common property to decide if an element belongs or not to the set). But the idea of absolute equivocity would be that of a non-relationship, no connection whatsoever, which is in some way contradictory. If we contrast the differences, say, of two sets, they must at least share a space, which makes that contrast possible.

Now, analogy should lead us through the path of *community* without unity and without identity, and without the dead-end of absolute equivocity. This is the core of analogy: to think the common without resorting to inflexible concepts of unity, identity or totality. In other words, what we think under the term analogy should allow us to think of relation in general without an underlying absolute unity (*hen kai pan* as it is said in Greek) like a subject, the world or God; without a common divisor (i.e., a ground or absolute basis); and without any whole (i.e., in a mereology, or theory of wholes and parts, we could never achieve the last totality, where everything would find its determinate place).

We turn back now to the concept of transformation. It is the concept of mapping as transformation that will allow us to think of different levels of similarity between spaces. The concept of transformation allows us to evaluate sameness. But sameness depends also on certain *axioms*. For example: in geometry (example a), a rhombus does not have the same symmetry group of the square. But in topology (example b), a circle is equivalent to a square, for they can be continuously deformed into each other.

a) b)

Fig. 3.

A transformation is nothing but a mapping. Such a mapping may render identical objects, like in the case of the square and its symmetry group, as we have seen. But not every transformation keeps *all* properties of the original object. Mapping in topology may, for example, involve an immersion or an embedding. Embedding implies that a space is "contained" in another space; it is a subspace of it. For example, a *Klein bottle* is a manifold embedded in 4-dimensional space (R4). We can, however, immerse it in three-dimensional Euclidian space (R3), where we obtain, however, *singularities* (like self-intersections) that cannot be "faithfully" represented in R3. We "lose" information by passing from 4 to 3 dimensions. We have a similar case in map projections, i.e., projections of the sphere (S2) onto the plane (R2), from which we derive the different types of Earth maps.

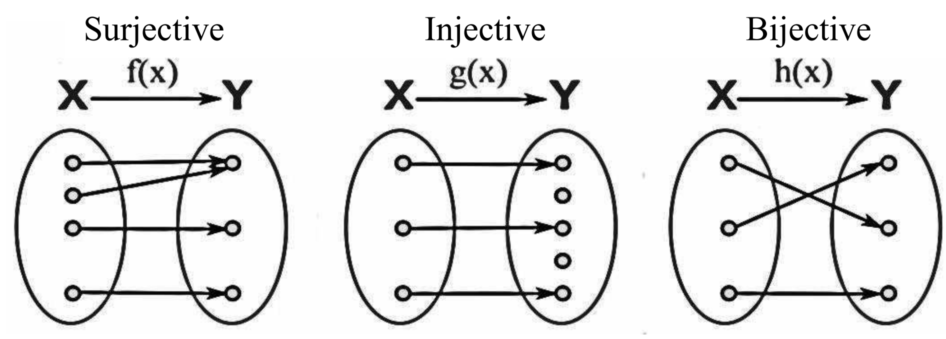

Fig. 4.[6]

[6] Wikimedia Commons under the license WTFPL 2.0 Source: https://commons.wikimedia.org/wiki/File:Surjection_Injection_Bijection-fr.svg"

In general, one can say that in the case of projections, we can have, on the one hand "better" or "worse" examples, depending on the function; and, on the other hand, different but equally good though only *partial* interpretations. Functions are ways to transform or to assign values of a set to another set through a rule. Functions may be, as we know, surjective, injective or bijective.

A bijection is the most faithful mapping, for it creates a one-to-one relationship of domain and codomain. This would render identity among two objects, and it would not be an analogy. But we have also equally "good" projections if they are all surjective. Non-surjective mappings (injective) are less accurate, because they are farther from the original object than in the case of surjective ones. This is a first glimpse of analogy understood in terms of functions or mappings. In the second case, we may have different surjective projections, all of them equally good, but they cannot be transformed one into the other, since each of them implies a *decision* on what to represent. Some projections respect one feature of the original object, some projections respect others.

5. Symmetries and diagrams

> What pattern connects the crab to the lobster and the orchid to the primrose and all the four of them to me? And me to you? And all the six of us to the amoeba in one direction and to the back-ward schizophrenic in another? [Bateson, 1979, p. 8].

All this may sound either too-mathematical or too abstract to have any philosophical salience. In order to extract far-reaching consequences from mathematical concepts when dealing with analogy, we need to comply with certain criteria. In the case of topology we have seen that to apply the notion of a map, we need sets or, even better, spaces, which are structured sets (i.e. they have a certain topology). Our philosophical concepts cannot be punctual, but have the need to belong to a net of relationships or to constitute a structure in themselves. Not only relationships between objects are needed, but between sets of objects with some structure, or even better, between structures.

In this section we direct our efforts to exploring whether geometry helps us to think analogically, focusing on one geometrical property, namely symmetry. To achieve this, we will now concentrate in the classical philosophical-ontological schemes advanced in modern thought, namely that of subject-object, and that of subject-subject. Such relationships are crucial not only for epistemology, but also for ontology and ethics.

The relationship Subject-Object could be represented with two letters (corresponding to each element) and a dash (representing some relationship: S-O). Since it is a reciprocal relationship, we could write it with a double arrow: S↔O. The relationship is not directed, so we could also invert our formula like this: O↔S. We can thus conclude it is symmetrical in a *general* sense. Let us consider now three elements: two subjects and one object. This would be the minimal depiction of the existence of *one* world with more than one perspective. This is also the minimal depiction of an "intersubjective" world. Intersubjectivity does not have here the form of absolute unity, but the subjectivity is "distributed". We speak of intersubjectivity because there are not many possible worlds, all indifferent to each other, but only one, which, however, may be seen from more than one perspective.

Two objects are symmetrical if one can continuously transform one into the other. Above, we saw already the example of the triangle. We could interpret rotations as the different possible *perspectives*, from which the object "looks the same":

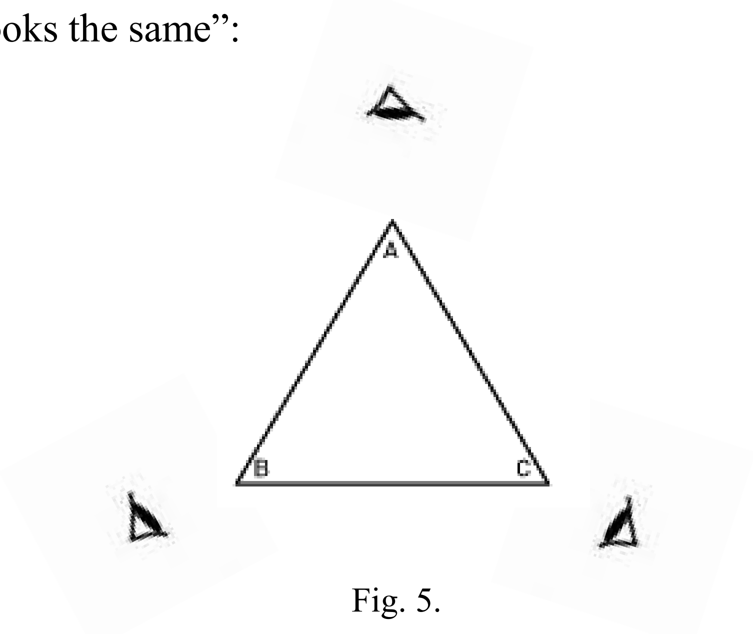

Fig. 5.

But we can also think of perspective in the sense that the Renaissance painting did, namely as points of view of the "same" world, rendering different views of it, as the following examples show:

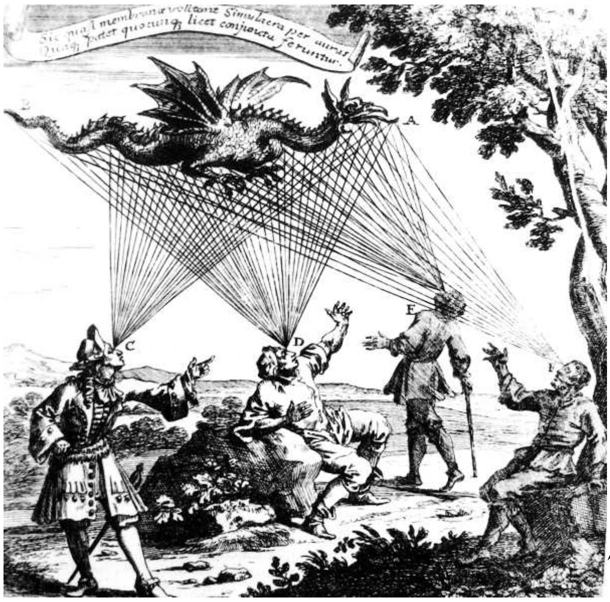
[7]

Fig. 6.

Now, we could represent the relationship between two subjects (S1, S2) and one world (O) in two different ways: linear (S1-O-S2), (S1-S2-O), (S2-S1-O), (S2-O-S1), (O-S1-S2), (O-S2-S1); or on a surface:

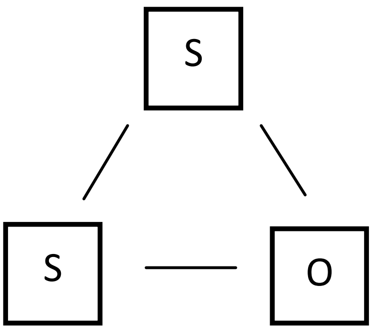

Fig. 7.

In the linear depiction S1-O-S2, we have a relationship between S1 and S2 only through the world, but it is not direct. It is, however, symmetrical

[7] Johann Zahn, "the radiating eye" from Oculus Artificialis Teledioptricus Sive Telescopium (1702). https://spyurk.am/uploads/images/scaled_full_4d38cd0d3554f455473b.png Consulted: March 20 2016.

to the form: S2-O-S1. In this sense, we have only three different linear possibilities: (S1-O-S2), (S1-S2-O), (S2-S1-O). The two last options are really not examples of intersubjectivity, for in both cases one subject has (indirect) access to the world, i.e. only through the other. In pure formal terms (S1-S2-O) and (S2-S1-O) represent the "same" case: indirect access. So, we could say that we really have two options: (S1-O-S2) and [(S1-S2-O) or (S2-S1-O)].

In the case of the plane depiction we can represent more structural elements. Actually, *all* combinations of the linear depiction are possible *simultaneously* in the planar one. It is just about writing the corresponding *arrows*. We are interested in very specific arrows. In our scheme we can ascribe different properties to the subjects and to the object at stake. Subjects can make interpretations of the world, but the world can´t make interpretations of them. It affects them, but in a different way:

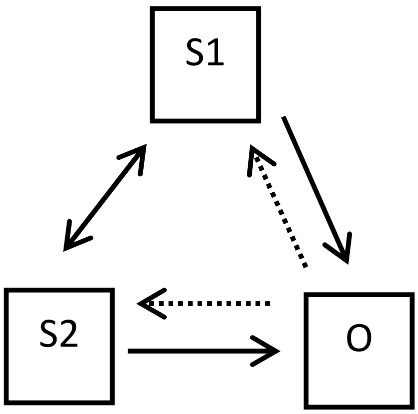

Fig. 8.

In our scheme, the arrow that goes in two directions means that S1 and S2 have a reciprocal relationship. We also see that S1 and S2 have a different but equivalent relationship to O. And finally, there is a relationship between every subject and the world, but this is not symmetrical in the same sense of the direct relationship between S1 and S2. In this manner, we can observe interesting symmetry properties. The *relationship* S1 to O and S2 to O are symmetrical in one sense, they are two "interpretations" of the *same* world. And yet, it is not exactly the same, for they render different perspectives.

The relationship S1 to S2 and S1 to O are not symmetrical. We can say that even though S1 and S2 are not the same, they have the same *relation-*

ship to the world. S1 is not S2, they are two different perspectives. But they are perspectives, and in this sense, they are the same, but only when contrasted to the world (O), which has no perspective at all.

In terms of negations, we can state that, from the perspective of S1, it is an *I*, and S2 is a *not-I*. But this not-I is also an *I*. Yet, O means also *not-I*, and that is true for both S1 and S2. We have, then, two types of negation, which in turn can be interpreted one as symmetry and one as asymmetry. The whole triadic complex is then a combination of symmetry and asymmetry. We recognize this in the distribution of three logical places: *I*, *you* and *it*. It is further true that both *I* and *you* are a part of the world; they emerge objectively from it, but they cannot be reduced to their objective existence. It is also true that for an *I*, the *you* and the *it* are *not-I*, but they have to be distinguished. The objective other and the subjective other do no coincide. Ontology (relationship Subject-Object) and ethics (relationship Subject-Subject) are different and yet interwoven. And it is lastly also true that although the *I* and the *you* have "access" to the same world, their views are not the same, otherwise we could collapse them into a single *I*, falling again into subject-object dualism. There is symmetry in the first sense (having access to the world), but also asymmetry, which so far both interpretations do not cover completely. This would mean that their interpretations are "analogous", but neither identical, nor absolutely different.

The German logician Gotthard Günther offers a reading of this triadic structure as we can see in the following scheme[8]:

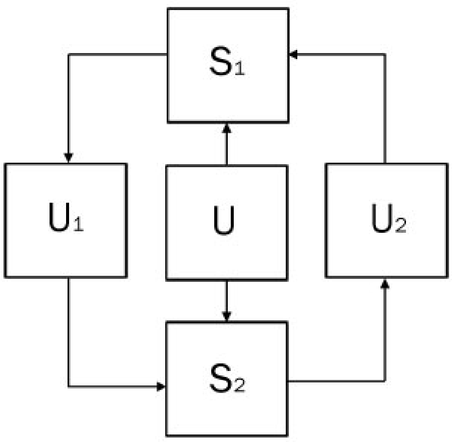

Fig. 9.

[8] After a diagram found in: [Günther, 1980, p. 88].

S1 and S2 are two subjective points of view of the world (U); the corresponding interpretations of the world are U1 and U2. The arrows show an "action" of the world on both subjects, who in turn generate an interpretation of it, which, in turn is "communicated" to the other. This diagram is interesting as it shows the logical distribution we have been talking about. We could now complicate the original scheme by introducing an element between pure subjectivity and pure objectivity (which by the way are only ideal poles in a multipolar structure), between both "I" (S1 and S2 included) and "it", for example, an animal (A in the diagram). An animal is not a pure object, but it is not a subject in the sense a human mind is.

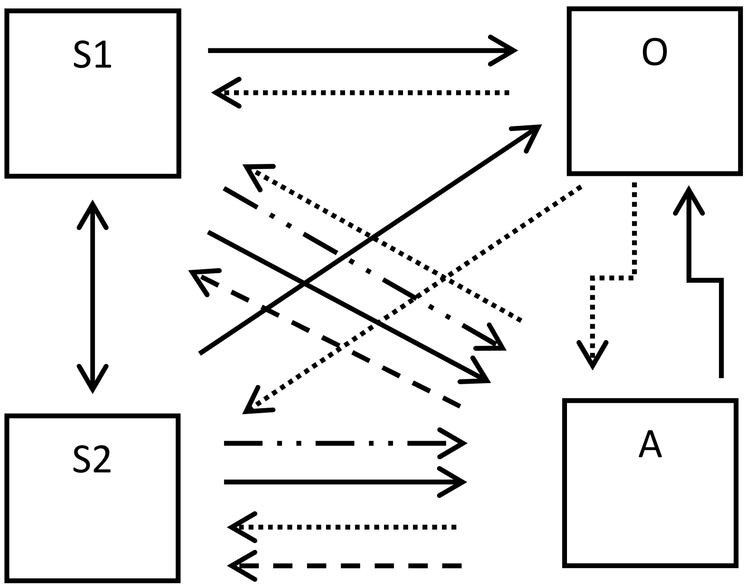

Fig. 10.

In the diagram the solid double arrow between S1 and S2 means a subjective relationship. Each subject (S1 and S2) has a double asymmetrical relationship to the world (O): the solid line that goes from subject to object means an objective apprehension; the dashed line means how the world "affects" the subject. Each subject has a triple relationship to the animal. First we have a solid line going from the subject to the animal, which means that it is an object of nature, like any other. The dotted line means how the animal affects a subject as an object. So far we have the same lines as in the relationship subject-object. But there is surely a specific way of being affected by an animal (represented by a dotted line) and a specific way to relate to it (represented by the dashed-dotted line). There is in some way a symmetrical relationship between all living beings, for they establish systems of energy transfer. But this symmetry does no reach so far as the symmetry between human beings. And even tough animals and humans exhibit an asymmetrical relationship; this is more symmetrical than the relationship between the living and the non-living. And the animal (that of course, we also are, but not only) has also a peculiar relationship to the world: perceiving and being affected by it, which we represent by the not-straight dotted and solid lines.

If we enrich this scheme with other regions of being, we could see a complex system, in which we can establish different "observers" and different "objects" (which can exchange their functions or positions), different negations and different orders and levels of symmetry. This, however, leaves us with some sort of hierarchy, just as if symmetry was lost as we go farther and farther from human subjectivity. If it is true, as we said at the beginning, that a) being is community, that philosophy as metaphysics is somehow a *koniology*, a *communology*, and that b) analogy creates a nexus between non-strictly related beings, then we should explain how the connectivity of being changes through analogy.

6. Connectivity

Aristotle´s ontology, we know, is inseparable from his idea of grammar and logic. The structure of language mirrors the structure of being, and both obey the broader structure of logic. Being in this sense is a predication in the sense of *apophansis*: showing the subject in the predicate or the substance in its attributes. Being is expression. But all the expression of singular beings is linked to being as such, to being-qua-being. We know that being is said in many ways (*pollachós*), but also that being adopts the structure (or difference) between genres and species. Even if we take substance (*ousía*) to be the last instance considering being qua being, the question remains open about what makes *community* in being, why and how all beings are gathered in being in general, instead of constituting disjoint atomic existences.[9] Aristotle´s division of being along the lines of genre and species produces a tree-structure like the following:

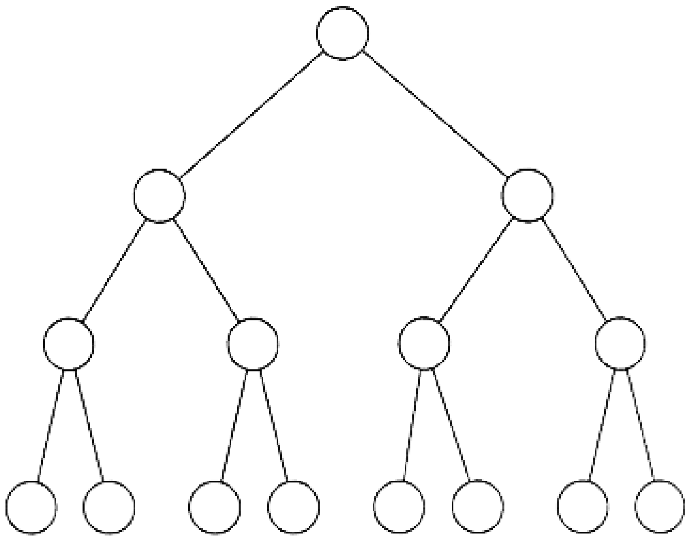
[10]

Fig. 11.

Being is *not* the top node in the reticular structure, but the *whole* structure, i.e., the *division of the one into many*. Today we see this tree in Linnaeus' classification of the living and in the diagrams of evolution. But how are elements related to one-another? Two horses, for example, are related so far as they belong to the same category, i.e., both are elements of the same set. A horse and a donkey may be related through a more general (an *upper*) category including, say, four-legged animals; and a man and a horse through the *higher* category of "animal". To relate different species we always need the upper category; because the absolute *connectivity* of the whole tree-structure depends on the unit at the top, i.e., it is absolutely *hierarchic*. It is only because of the top unit (the *One*), that all branches remain communicated. This structure of beings relies, of course, on its

[9] Stéphane Dugowson [2012] offers interesting contributions to define "connective spaces" from the standpoint of category theory.

[10] This analysis draws on ideas presented in [Günther, 1980]. The main focus of Günther is however a many-valued logic, without a reflection on the more general issue of connective structures.

binary logic and its unique negation. Every node divides into two, corresponding to the A and not-A form. Now, if we descend very low down the pyramid and try to establish a relationship between two species, we then have to climb up in the tree-structure until we find a *common category* which includes them.

There are some non-classical logics like that of Gotthard Günther (which he called *polycontextural*) that suspend the axiom of Aristotelian logic: *tertium non datur.*[11] In this case, we have not only two values (true, false), but a third one. This third value, however, can work again in a two-valued structure, but in another "contexture". It is as if the whole space of "being" was constructed by *patches of Aristotelian-logic* very much like non-Euclidian geometry (or a variety in general, to put it in Riemann´s terms) is constructed by patches of Euclidian-space. The structure of logic, i.e., its "valuedness" determines the *connectivity* structure of being. In this sort of lattice, we can connect beings through different *paths*, which do not have to go up in the structure like in the case of Aristotelian logic. These sort of lateral connections are not hierarchic anymore, but so-called *hetararchic.*[12]

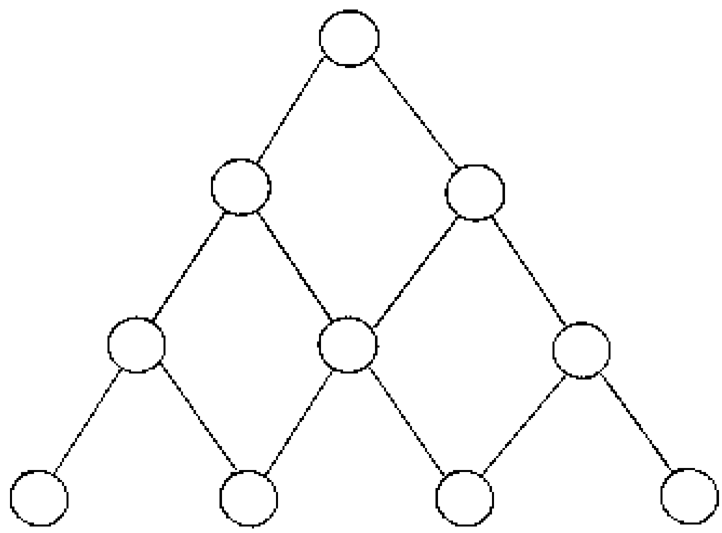

Fig. 12.

[11] See: [Günther, 1979] and [Günther, 1980].

[12] The concept was coined by Warren McCulloch in his seminal work on neural networks: [McCulloch, 1945]. Heterarchy implies lateral connections, which complement hierarchic ones.

In terms of negations we have, for every "opposition", two more options: the "and, and" (A and not-A); and "neither-nor" (neither A not not-A). This gives us an extended square of logical places, as Kaehr, a pupil of Günther suggests[13]:

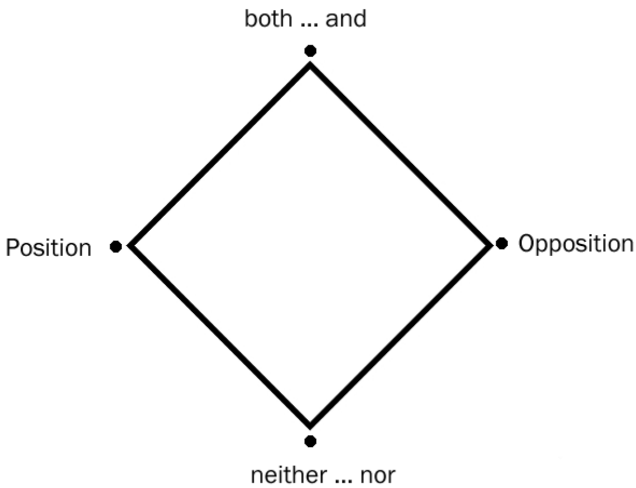

Fig. 13.

Now, to construct the whole polycontextural pyramid of being, que, we have to iterate the fundamental square in such a way that they cover the whole space, like in a *tessellation*.

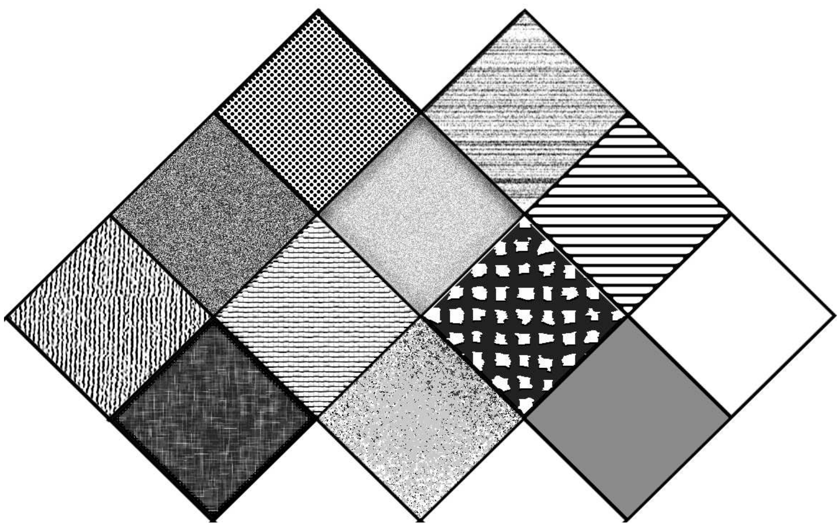

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Fig. 14.

[13] After [Kaehr, 1997, p. 20]. This corresponds by the way to a classical Indian square of oppositions, the so-called catuskoti.

[14] After [Kaehr, 1997, p. 20].

We should note, however, that the "whole" space is not simply connected; it is not a homogenous space. The "whole" is rather constructed through "patches" of this fundamental square. This is the main idea of polycontextural logics: for every context, we have an Aristotelic "world", plus two "unnamed possibilities". These two possibilities: (both, and), (neither, nor) remain unnamed in the same contexture, but obtain a positive content in some other contexture. Important is to separate the "and, and" and the "neither, nor", for even if both reject the opposition, they do it in different ways.

It is not said if this structure is "flat". It might well be the case for such a geometric structure of logic to be three or more dimensional. This is a fundamental question, because we can pose here the question about the continuity and discontinuity of different contexts (or contextures) and more radically, of logic. In other words, we ask if logic can be applied to a homogeneous world. In topology we distinguish between continuous (connected) and discrete (non-connected) spaces. But within continuous spaces, it is still to decide *how* they are connected. A space with a hole, for example, is non-simply connected. But there are other spaces, called multi-connected, which offer a combination of continuity and discontinuity. An example of such spaces is a polyhedron, which is connected, but at the same disconnected through the edges. If we construct a polyhedron for logic, we could not escape such questions.[15]

We have seen how logic changes *connectivity*. And here I come to our main issue again: analogy. I have argued that analogy changes the connectivity of being. To explain this, I resort very briefly to Husserl´s concept of the world as a *horizon*. The world of the horizon will show itself to be articulated by what Husserl calls "lines of analogy".

[15] The work of Jean-Yves Béziau is a remarkable example of a "geometric" interpretation of logic, both in the planar form of an hexagon and in a three-dimensional construction. In this sense, paraconsistent logics and the extension it provides to classical approaches offer the possibility to link logics with mathematical structures in the sense of Bourbaki. See [Béziau, 2001].

7. Analogy and phenomenology: the world

In his late writings, Husserl developed the concept of the *world* as an indeterminate and open horizon that serves as a background (*Hintergrund*) for all our explicit conscious thinking, which works as a foreground (*Vordergrund*). But very early, in his *Lessons on the phenomenology of time-consciousness*, he writes: "[…] if we have in the [temporal] succession unequal objects with equal distinctive moments, then certain "lines of equality" [*Gleichheitslinien*] run from one to the other, and in the case of similarity, then lines of similarity [*Ähnlichkeitslinien*]. We have here a reciprocal relationship [*Aufeindanderbezogenheit*] that does not constitute in [explicit] consideration, but lies at the base, as a presupposition for every intuition of equality and difference [*Gleichheitsanschauung und Differenzanschauung*] before every "comparison" and every "thought" [takes place]" [Husserl, 1966, p. 44].

In *Experience and Judgement*, Husserl clarifies this idea of a pre-predicative sphere as indeterminate and open. He states that we have a pre-theoretical approach to the world, a *pre-knowledge* (*Vorwissen*), which is, regarding its content, "indeterminate or incompletely determined, but never empty" [Husserl, 1939, p. 27]. This experience constitutes, further, an "experience horizon", that allows us to determine a thing ever more and more without ever exhausting it. It is rather our interests and goals in the world that lead us to say: it is enough, this degree of determination and detail suffices. Husserl concludes the following:

> I can convince myself that no determination is the ultimate, that the effectively experienced always has an infinite horizon of possible experience of itself. And this [horizon] is, in its indeterminateness [*Unbestimmtheit*], and in advance, in co-validity [*Mitgeltung*] as a space of possibilities [*Spielraum von Möglichkeiten*], hinting at a path of closer determination, that only in real experience is decided for a determinate possibility, actualizing it against other possibilities [Husserl, 1939, p. 27].

Our pyramid of being should now look like this:

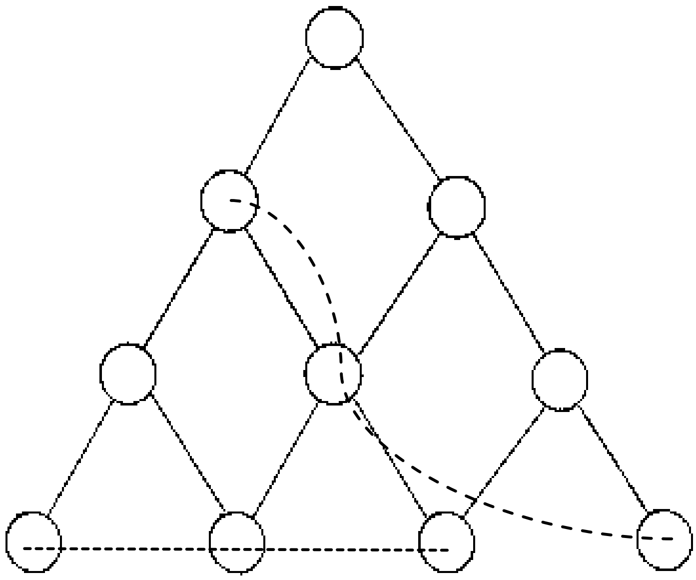

Fig. 15.

This means that before we constitute explicit objects and explicit relationships among them, there are lines of similarity, of analogy, of potential objects that run along an indeterminate horizon. It is as if objects existed only in certain virtuality, still full of possibilities, even contradictory, for we cannot apply our logical rules of identity and difference. The horizon is, let´s call it for the moment: polyvalent, or *paraconsistent*. As a metaphor we could remember here Schrödinger´s mental cat-experiment. Before we see inside the box, since the being-alive or being-dead of the cat depends on the spin of the electron, and since before observation, the spin is not determined, then the cat is at the same time dead *and* alive. Within the horizon, multiple possibilities dwell, until an object is determined and therefore becomes an explicit subject for conscious consideration.

We have the extended non-classic pyramid, but we have opened now strictly horizontal lines (paths) of analogy between beings. Analogy means here a non-hierarchical (i.e., a *heterarchical*) relationship. Dotted lines represent those possible lines of virtuality, characterized by non-classical logic but also by many types of morphisms and symmetries. Now such lines appear as possible *paths* in the structure of being. Husserl characterized the pre-scientific world as qualitative, vague and open. This is precisely what group-theory and topology provide us with: a systematic way of reasoning within the *qualitative*. Rotations, reflections, point symmetries, self-similarity, patterns in stacking and packing, progressions in series, strange attractors (or symmetries within chaos), fractals, etc., all this world is built upon qualitative similarity, that we find in the world as pattern and as a mixture of symmetry and asymmetry, before we construct rigid categories, structures, totalities and absolute unities of knowledge. It is the freedom within order itself, flexibility without arbitrary similarities, and it is the *connectivity* of beings along the dividing lines of categories. It is not the world before the world, but another layer, based on non-metric features, but in analogy.

8. Concluding remarks

I claim that if philosophy is to be understood in some systematic form, the central notion must be that of *structure*. Structuralism in mathematics, especially that of Bourbaki, tried to define "mother structures", like algebraic, order and topological ones. Philosophy and the social sciences adopted in course of the 20th century fundamental ideas of the Borubaki group. Structuralism should not be interpreted as a unified approach, whose fundamental concepts were established once and for all. On the contrary, there is no univocal and established concept of structure. We can say however that structures are sets (i.e. there is a "multiplicity" at stake) with some additional order. The contrary of order, so-called disorder, can be seen as triviality. Disorder is not "chaos" or "chance", but indifference. In thermodynamics disorder is called entropy. We see a phenomenon of entropy for example in heat-diffusion between two bodies. There order, and therefore difference, when two bodies in contact have different temperatures. We witness then a diffusion of heat, so that after a period of time, both bodies in contact have the same temperature. They cannot be differentiated anymore from that point of view. In nature, order can be seen as a long-lasting stability, or as *structural stability*. Such stability is not outside time, nor responds to eternal patterns. We can think of dynamic or relatively static forms of order, but in any case we have always *different elements* and some *possible operations* between them, what produces a sort of "space of possibilities" in the wide sense of the word.[16]

Philosophy, logics and mathematics have conceived different types of individuals (what counts as an element of a set), of differences (and consequently, of negations) and of relationships between elements. At the beginning of the paper we spoke of univocity and equivocity. They are two modes of establishing mappings between spaces. Some words, for example, may have one or multiple meanings. In this sense, analogy can be seen as a) a way of establishing relationships between elements that do not belong to the same class (neither univocity, nor equivocity) and b) a way of establishing mappings between different spaces in different senses *at the same time*. It is as if analogy confronted us with another way of conceiving difference and thus of conceiving orders and structures in general. Structural mathematics, but also non-classical logics have opened new worlds that allow us now to redefine our old concepts on order and being in general. In mathematics: we have highly counterintuitive concepts of continuity, discontinuity, limit, interiority, exteriority, infinity, cardinality ("size" of sets), etc.; and in non-classical logics, we have new conceptions of negation and opposition. All this findings point to what we could name a new sort of "philosophical logic". But we are just starting the long and strenuous task of linking anew philosophy, mathematics and ontology.

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[16] Luciano Boi has also shown a mathematical approach to ontological problems through the consideration of topological spaces. See his remarkable book the *Morphology of the Invisible* [Boi, 2011], which has inspired many of the ideas of this paper.

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Arturo Romero Contreras
Benemérita Universidad Autónoma de Puebla, Mexico
rcarturo@gmail.com