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Models and their Foundational Framework

Abstract. The term model is mainly used in two meanings which are considered to be different: a model of a problem domain as a conceptualisation; a model of a set of formulas as an interpretation in which every formula within this set is true. A general theory of models has not yet been developed. H. Stachowiak proposes a phenomenal approach and 'defines' models by their properties of mapping, truncation and pragmatics. Meanwhile, a notion of the model has been developed. At the same time, it seems that there are rather different understandings of model in sciences and especially Mathematical Logics. Sciences treat models as reflections of origins. Mathematical logics considers models as an instantiation in which a set of statements is valid. So, mathematical model theory is often considered to be a completely different approach to modelling. We realise however that mathematical model theory is only a specific kind of modelling. We show that the treatment of models in logics and in sciences can be embedded into a more general framework. So, the theory of models is based on a separation of concern or orientation.

Keywords: models, mathematical model, instrument, framework, unifying theory.

1. Introduction

Modelling is a topic that has implicitly been in the center of research in science and engineering since its beginnings. It has been considered as a side issue for long time. During the last 40 years it has gained more attention and becomes nowadays a subdiscipline in many disciplines. The compendium [TN15] introduces models in agri- culture, archeology, arts, biology, chemistry, computer science, economics, electrotechnics, environmental sciences, farming, geosciences, historical sciences, languages, mathematics, medicine, ocean sciences, pedagogical science, philosophy, physics, political sciences, sociology, and sports. The models used in these disciplines are instruments used in certain scenarios. So, essentially it is an old subdiscipline of most natural sciences with a history of more than 2.500 years [Mu¨ l16][1]. It is often restricted to Mathematics and mathematical models what is however to much limiting the focus and the scope.

The *modelling method* is a specific science method that uses models as instruments with certain intention or goal, e.g. for solving a task. The model represents or deputes origins. The model is used instead of the origin due to its properties, esp. adequacy and dependability. The modelling method thus consists (i) of the development of 'good' models, (ii) of the utilisation of the model according to the goal, (iii) of the compilation of the experience gained through model utilisation according to the goal, and finally (iv) of generalisation of the experience back to the origins. So, a model must be well-build for this goal, must be enhanced by methods that support its successful deployment, and must support to draw conclusions to the world of its origins.

1.1. A Model is an Adequate and Dependable Instrument

A **model** *is a well-formed, adequate, and dependable instrument that represents origins* [Tha14, Tha17a].

Its criteria of well-formedness, adequacy, and dependability must be commonly accepted by its *community of practice* within some *context* and correspond to the *functions* that a model fulfills in *utilisation scenarios*.

As an instrument or more specifically an artifact a model comes with its *background*, e.g. paradigms, assumptions, postulates, language, thought community, etc. The background its often given only in an implicit form. The background is often implicit and hidden.

A well-formed instrument is *adequate* for a collection of origins if it is *analogous* to the origins to be represented according to some analogy criterion, it is more *focused* (e.g. simpler, truncated, more abstract or reduced) than the origins being modelled, and it sufficiently satisfies its *purpose*. Well-

[1] The earliest source of systematic model consideration we know is Heraklit with his λογος (logos). Model development and model deployment is almost as old as the mankind, however.

formedness enables an instrument to be *justified* by an empirical corroboration according to its objectives, by rational coherence and conformity explicitly stated through conformity formulas or statements, by falsifiability or validation, and by stability and plasticity within a collection of origins. The instrument is *sufficient* by its *quality* characterisation for internal quality, external quality and quality in use or through quality characteristics such as correctness, generality, usefulness, comprehensibility, parsimony, robustness, novelty etc. Sufficiency is typically combined with some assurance evaluation (tolerance, modality, confidence, and restrictions). Model functions determine which justification is required and which sufficiency characteristics are important. A well-formed instrument is called *dependable* if it is sufficient and is justified for justification properties and sufficiency characteristics.

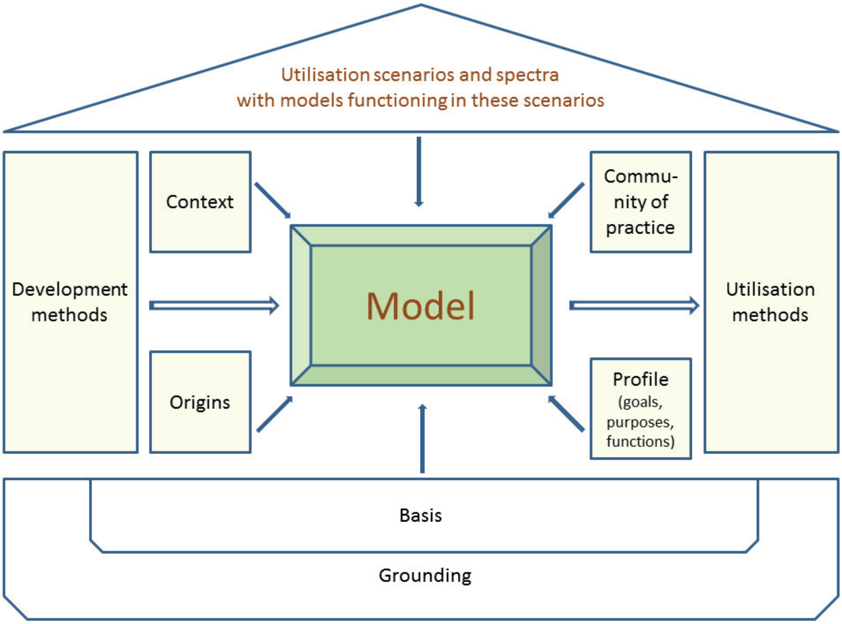

Figure 1: The model as an instrument that is adequate and dependable for its driving directives (origins, profile (functions, purposes, goals), community of practice, context) within its background (grounding, basis) and that properly functions in utilisation scenarios as a deputy of its origins

Figure 1 represents a model of the model. The development and utilisation methods form the enabling aspects of the modelling method. Driving directives are (1) origins to be represented by the model, (2) purposes or goals or functions of models[2], (3) the community of its users and developers, i.e. the community of practice, and (4) the context into which the model is embedded. Models function as instruments in application or utilisation scenario. Typical functions of models are (a) cognition, (b) explanation and demonstration, (c) indication, (d) variation and optimisation, (e) projection and construction, (f) control, (g) substitution, and (h) experimentation. A model is not built on its own. It has an undisputable grounding that has to be accepted. The basis of the model - similar to the cellar - can however be disputed. Grounding and basis form the background of a model. We observe that the background is often given only in an implicit form. The same kind of concealment can also be observed for the utilisation scenario which are implicitly given by sample and generalisable case studies for the utilisation frame.

The model is not simply an image of its origins. The mapping property [Kas03, Mah09, Mah15, Sta73] might be too restrictive for models. Instead, we use analogy. Models can also be material artifacts, e.g. exhibition models, architecture models, models used in religion, and three-dimensional demonstration models used in engineering and mathematics [CH04]. A model can be a model of another model. For instance, the topographical Königsberg bridge sketch is a model that is the origin for the graph-theoretical model for the Euler path existence [MV10]. Models might follow different structuring

[2] Goals, purposes, and functions are often considered to be synonyms. We follow the separation of concern as discussed in [TN15]. The *goal* of a model can be defined as a ternary relation between initial states, final states and the community of practice that accepts the final state and considers the initial state. It describes the aim, the ambition, the destination, the end, the intent, the intention, the objective, the prompt, or the target of a model. It is not of interest whether the goal is realistic. The *purpose* of a model extends the goal of the model by means (or instruments) that potentially enable the community of practice to achieve the goal. The *function* of a model embeds the model into practices and scenarios of its application. The function can be considered as an extension of the purpose by `application games' of the model. It specifies the role and the play of a model in the scenarios, i.e. how, when, for which/what or why, at what/which etc. the model functions. The function often implicitly adds conventions of deployment, customs, exertions, habits, specific usage and uses, and handling pattern to the purpose. The model thus functions in the scenarios in the given mould.

and behaviour than the origins. For instance, many models in classical physics concentrate on aspects and do not represent reality, e.g. the Bohr atom model. The Ptolemaic world model has a complete different behavior for most of the bodies of the universe but is was useful (see the nautical tools used as that time or the clock of Antikythera). The hydrological model of the electrical circuit has an analogical behavior that is not based on a mapping. Usefulness and utility according to goals govern the selection of a model instead of quality characteristics such as validity. Finally, a model comes with its background. It cannot be properly understood and used if the background is concealed. Let us distinguish the concepts of goal, of purpose, and function in the sequel. The goal of a model is in general the association between a current state and the target state that is accepted by stakeholders or – more general – by members of a community. The purpose enhances the goal by means that allow to reach the target state, e.g. methods for model development and utilisation. The function extends the purpose by practices or – more systematically – by scenarios in which the model is used. A typical scenario is the modelling method and its specific forms.

1.2. Models in Science and Daily Life versus Models in Mathematical Logics

Models in sciences and model theory in mathematical logic are often considered to be completely different issues [Bal16]. This point of view is correct as long there is no consolidated understanding of a notion of a model. Models in model theory are instantiations of a set formulas. This set of formulas is satisfied by a model according to a logical definition frame. The model is a structure that is defined with the same signature as the set of formulas.

So, we might come to the conclusion that there are at least three different understandings of the model. We will oppose this conclusion in the sequel. It is only true for the Fuzzy or phenomenalistic view.

Models in science typically follow the modelling methods. They may be composed of a number of models and be based on other models. A model must not be true. It should however be coherent to some extent within its discipline

The origin in science is not limited to material origins. The origin itself can be virtual or be again a model, e.g. a mental model. So, the modelling methods may also be iteratively applied.

Models often used in daily life. One kind are metaphors or parables. The typical kind is, however, a pattern for explanation, negotiation, and communication. Models carry a meaning. It is often debated whether a fashion model or a diagram or a visualisation can be considered as a specific kind of a model.

The modelling method presented so far is associated with its origins. We might however also use models for construction of other origins or models. In this case, the model is not generalised but used as a blueprint for another artifact. So, we observe that the modelling method must be extended.

1.3. Models and their Utilisation Scenarios

Models are used in various scenarios, e.g. communication, system construction, perception, analysis, forecasting, documentation, system modernisation and optimisation, control, management, and simulation. Let us in the sequel concentrate on the first three scenarios.

The extended modelling method is embedded into a more general form of activities, i.e. scenarios. The model itself is used as an instrument in a scenario or a bundle of scenarios which we call usage spectrum. It has a function or a number of functions in these scenarios. This functioning must be effectively supported by utilisation methods and is used by members of a community of practice in most cases. For instance, models of situations/states/data are often used for structuring, description, prescription, hypothetic investigation, and analysis. So, we observe that the function (or simpler the purpose or the goal) of the model is determined by the concrete way how a model is used.

A model might be oriented towards this community of practice. It can however also represent the scenarios themselves. It might represent the context of these scenarios, e.g. the scientific or engineering background, the relation to time and space, the application area insight, and the knowledge accepted by the community. It might also be oriented to representation of either a situation and state under consideration or a evolutionary change process.

The different orientations are the basis to distinguish the six concerns for models: community of practice, back- ground/knowlegde/context, application scenario and stories of model utilisation with their specific frames, situation/state/data, dynamics/evolution/change/operations, and models as representations and instruments. Figure 2 shows the relation between the concerns and the functions a model might have[3]

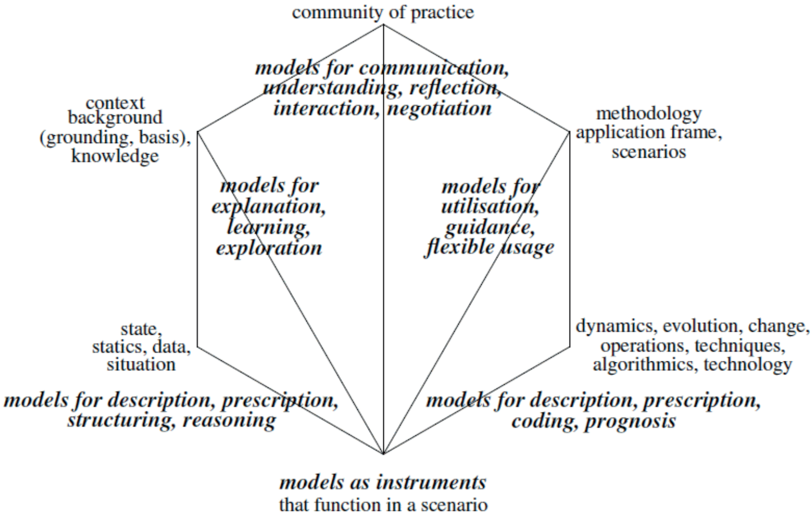

Figure 2: Models and the five concerns in model-based reasoning, investigation, and engineering

1.4. The Storyline of the Paper

A general theory of models, of modelling activities and of systematic modelling has not yet been developed although modelling has already attracted a large body of knowledge and research[4]. The notion of the model is not yet commonly accepted. Instead we know a large variety of rather different no-

[3] Modified and revised from [Tha17c].

[4] It is not our purpose to develop a bibliography of model research. Instead we refer to bibliographies in [TN15] and the more than 5.000 entries in R. Müller's website, e.g. [Mül16].

tions. Model development activities have been a concern in engineering. The process of model development has not yet attracted a lot of research. Model deployment also needs a deeper investigation. The model is mainly used as an instrument in certain application scenarios and must thus function in these scenarios. So, a model is a medium.

We have already introduced the general notion of a model as a starting point. The next step could be the development of a general theory of modelling. It is often claimed that modelling is rather different in science and engineering. So, we might conclude that there is no general theory of modelling. This paper is going to show that there is a general theory of modelling. We start with a case study in Section 2. These lessons gained in this cases study are a starting point for a general theory of models, of modelling activities, and of systematic modelling. In Section 3 first elements of this theory are developed.

2. Models in Everyday Life and Sciences: A Case Study

Analysing model notions we realise that there are at least four different approaches:

1. The general phenomenalistic definition uses properties such as mapping, truncation and pragmatic properties for the association between origins and models. Most research on models starts with this approach.

2. The axiomatic definition follows frames used in Mathematical Logics and defines models as exemplifications of formal systems and formal theories. Models thus depute and represent a certain part of reality.

3. The mapping-based definition is based on a direct homomorphic mapping between origin and model. We might have another mapping between model and implemented system that is a realisation of the model.

4. The construction-oriented definition defines a model as being a result of a modelling process by some com- munity of practice.

There is a fifth approach to models which simply uses artifacts as models without any definition, e.g. in human communication and also in sciences[5].

[5] One of the prominent definition is given by John von Neumann [vN55]: "*The sciences do not try to explain, they hardly even try to interpret, they mainly make models. By a model is*

The definition given above follows, however, the mathematical way of defining things through definitional extensions.

Models are used as

- *perception models* reflecting someone's understanding,
- *mental models* that combine various perception models and that make use of cognitive structures and operations in common use,
- *domain-situation models* representing a commonly accepted understanding of a state of affairs within some application domain,
- *experimentation models* that guide experimentation,
- *formal models* based on some kind of formalism,
- *mathematical models* that are expressed in some mathematical language and based on some mathematical methods,
- *conceptual models* which combine models with some concept and conception space,
- *computational models* that are based on some (semi-)algorithm ,
- *informative models* that used to inform potential users about origins,
- *inspiration models* that provide an intuitive understanding of something,
- *physical models* that use some physical instrument,
- *visualisation* models that provide a visualisation,
- *representation models* that represent things like other models,
- *diagrammatic models* that are based on some diagram language with some kind of semantics,
- *exploration models* for property discovery,
- *prototype models* that represent a class of similar items,
- *mould models* that are used for production of artefacts,
- *heuristic models* that are based on some Fuzzy, probability, plausibility etc. relationship, etc.

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meant a mathematical construct which, with the addition of certain verbal interpretations, describes observed phenomena. The justification of such a mathematical construct is solely and precisely that it is expected to work - that is correctly to describe phenomena from a reasonably wide area. Furthermore, it must satisfy certain esthetic criteria - that is, in relation to how much it describes, it must be rather simple. I think it is worthwhile insisting on these vague terms - for instance, on the use of the word rather. One cannot tell exactly how ``simple" simple is. Some of these theories that we have adopted, some of the models with which we are very happy and of which we are proud would probably not impress someone exposed to them for the first time as being particularly simple."

Although this categorisation provides an entry point for a discussion of model properties, the phenomenon of being a model can be properly investigated. Each category is rather broad and combines many different aspects at the same time. We already introduced a general notion of model. In this Section we will investigate whether the general definition covers all these kind of models for science and also daily life and whether it can be supported by a holistic treatment of models.

2.1. Models in Mathematical Logics

Let us consider only one kind of logics: classical Mathematical Logic based on first-order or higher-order predicate logics. Mathematical logics considers models as an instantiation in which a set of statements is valid. An arbitrary structure of the same signature as the logical language of the statements which satisfies this set of statements is called model or realisation of this set [Tar56]. Similar observations can be drawn for other mathematical logics as well. Mathematical logic has a long tradition of model research. Model theory became its branch and has a deep theoretical foundation. The main language is the first-order predicate logic. This language is applied in a rigid form [ST08] that became a canonical form of Mathematical Logics: It uses a canonical way of associating syntactic and semantic types. Additionally, the semantic type and the syntactic type have the same signature. The expressions of syntactic types are inductively constructed starting with some basic expressions of certain construct by application of expressions of some other construct. For instance, we may start with truth values and variables. Terms and formulas are then based on these basic expressions. The context is not considered. The world of potential structures is typically not restricted. The rigidity however allowed to gain a number of good properties. For this reason, first-order predicate logics became a first-class fundament for Computer Science.

A typical example of a standard set of logical formulas characterizing real numbers (commutativity and associativity of addition, existence of an additive identity and an additive inverses (0), commutativity and associativity of multiplication, existence of an multiplicative identity (1), existence of a multiplicative inverses with the exception of *the additive identity, distributivity of multiplication over addition, linear order). The set of real numbers with classical operations (+,*),predicates (=, <) and the numbers 0,1 as additive and multiplicative identity is the standard model of these logical formulas. Another model is the set of Robinson numbers which are used in a nonstandard approach to analysis based on infinitesimals. Infinitesimals are greater than 0 but smaller than any positive real number.*

In general, a model in Mathematical Logics is defined through its relationship to a set of formulas. These formulas are valid in the model. Additionally, axioms and rules of the first order predicate logics are valid in the model since they are valid in any structure of given signature. Models are thus instantiations (or exemplifications) for a set of statements. The theory of deduction is the main basis for reasoning. Therefore, the five concerns in Figure 2 have the specific peculiarity shown in Figure 3.

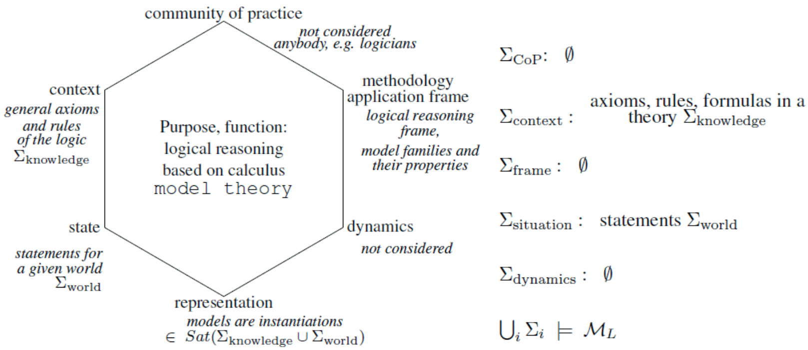

Figure 3: Models in logics for investigation of situations and expressible properties: axioms and rules form the context world; admissible states are characterised by a set of formulas; models are instances of potential systems that obey the system

The special side of the approach of Mathematical Logics to modelling is the consideration of the set of all potential models together with a given instantiation. This approach is however also taken into consideration for other model kinds as we shall in the sequel.

A model might become then an exemplar or prototype for a given theory. It can represent this theory and thus allows to reason on the given theory. It can be thus a final or an initial model (see the theory of abstract data types [Rei84, Wec92]) where the first one is the best and most detailed representation of the given theory and allows to reason on all potential negative statements as well.

We notice that classically the community of practice is not considered. Also, dynamics ist not an issue. There is not really defined any reasoning frame beside the calculus itself. We are free to choose Hilbert style or Gentzen style or any other derivation style for reasoning.

A specific decision within mathematical logics is the invariance of the signature, i.e. models as structures and logical languages for theory statements share the same signature. Therefore, there is a tight mapping between terms and formulas and the properties that can be stated on the model.

This specific mapping property has also been used for the phenomenal characterisation of models as structures that a based on a mapping from the origin to the model, e.g. [Bal82, Sta73, Ste66, Ste93]. We also observe that the truncation or abstraction property is a specific property of logical models.

2.2. Mathematical Models

Mathematical models are considered to be the most prominent kind of model. A mathematical representation of another 'donor' or *origin model* is based on the mathematical language. The mathematical model is used for solving of problems that have been formulated for the origin model. The association between the mathematical model and the origin model must be problem invariant. Solution faithfulness is often not given explicitly required, i.e. the solution obtained for the mathematical model must be faithful for the origin model. Mathematical modelling presumes the existence of this origin model. So, (1) it starts with an application analysis and a formulation of the problem to be considered in the application area. Next, (2) this formulation is transformed to the origin model which allows to describe the problem. (3) This origin model is then mapped to a mathematical model. (4) The fourth phase is the development of a solution of the problem within the mathematical model. (5) The solution is verified and will be validated for faithfulness within the origin model. Finally, (6) the solution is examined for its reflection in the application area. If the solution is not of the required quality then the phases are repeated. This 6-phase circular frame [GKBF13, Pol45] is a commonly accepted scenario for mathematical modelling.

A typical mathematical model is the Euler graph model for the Königsberg bridge problem (for detailed consideration see [MV10]). This graph represents a rough topographical model which is a model of the inner city of Königsberg. The purpose of the Euler graph is to provide a solution of the bridge path problem within a graph theory setting, i.e. providing a path – called Euler path - that uses each bridge once and only once. The solution to this problem is the condition that an Euler path exists if and only if the degree of all nodes is even except maximal two nodes with odd degree.

We observe that the mathematical frame is similar to the logical reasoning frame. Main quality requirements satisfaction of the problem solving purpose, adequacy of the mathematical model, robustness against minor changes, and potential and capacity for problem solution. The community of practice should not influence the model properties. It may influence on the selection of various representation models. The situation and its dynamics determines the appropriatedness of the mathematical language. The mathematical model is determined by some mathematical method that has shown useful in the past.

Our model notion extends the model discussion by H. Hertz [Her84, vDGOG09]. He postulates that some artefact is a model due to its analogy to origins, its dependence within an application context, its purposefulness, its correctness, its simplicity, and its potentially only implicit given background. Models have thus a validity area.

Mathematical models are specific formal models. They are based on a formalisation that can be mapped to some mathematical language. The mapping from the formal model to the mathematical model should preserve the problem, i.e. it is invariant for the problem. The mapping should additionally also allow to associate the mathematical solution to the problem with a correct or better faithful solution in the formal model and for the origins, i.e. the model is solution-faithful [BT15]. The mathematical language has not only a capacity and potential. It also restricts and biases the solution space. The calculus used for the derivation of the model is any mathematical and not restricted to logical reasoning.

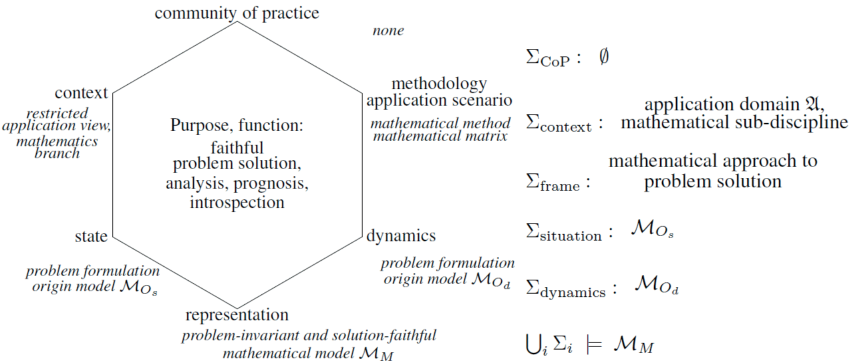

Figure 4: The mathematical model as a representation of a origin model within the mathematical frame

2.3. Science Models

All sciences widely use models. Typical main purposes are explanation, exploration, hypothesis and theory development, and learning. Models are mediators, explainers, shortcuts, etc. We can consider models as the third dimension of sciences [BFA+ 16, TD16, TTF16][6]. Following [Gra07], sciences may combine empirical research that mainly describes natural phenomena, theory-oriented research that develops concept worlds, computational research that simulates complex phenomena and data exploration research that unifies theory, experiment, and simulation. Models are an essential instrument in all four kinds of research. Their function, however, is different as illustrated in Figure 5 [BFA+ 16].

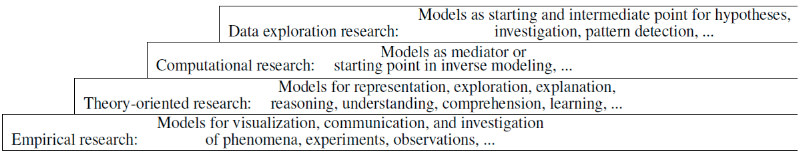

Figure 5: Some model functions according to the kind of scientific research

[6] The title of the book [CH04] has inspired this observation

Empiric research also uses a canonical modelling mould. Beside an ad-hoc mould we might use a sophisticated one: (1) define a research question (based, for instance, on the rhetoric frame (who, what, when, where, why, in what way, by what means (Hermagoras of Temnos, also Augustine's De Rhetorica) or W*H framework [DT15]), (2) consider threats to the research, (3) choose a research model (e.g. positivistic), (4) develop an approach how facts become theories, (5) create a generic meta-model (with some level of abstraction, with independent and dependent parameters and indicators), (6) define analysis approaches (qualitative or quantitative), (7) define the research pro- gram and agenda as a specific research process, (8) select the research method, (9) analyse the capacity and potential of quantitative data, (10) design the experiment, (11) design the case study, and (12) design the outcomes survey.

The empirical research approach often combines qualitative and quantitative approaches. The quantitative approach is often oriented on observable data whereas the qualitative approach orients towards theory, on concepts and conceptions, and on a characterisation of the situations of interest. The quantitative theories are often 'phenotypical' approaches contrary to the 'genotypical' approaches used in qualitative approaches. A typical approach is used in the collaborative research centre 1266[7]. It uses additionally an investigative reasoning approach. Figure 6 shows the differences between genotypical and phenotypical models. We use a planar representation of the three dimensions: (1) the composition dimension with sources, concepts, and theories; (2) the kind dimension with qualitative and quantitative reasoning, and (3) the model dimension that allows to concentrate on certain aspects of the first dimensions depending which function, purpose and goal the model should satisfy. A typical specific treatment of concepts is applied in modelling. Since models orient on certain aspects and represent also combined representations, concepts used in models are often not directly derived from concepts in the theory. Additionally, we should distinguish between quantitative, investigative, and quantitative models. The model kind in Figure 6 uses investigative reasoning and lends some elements from quantitative and qualitative theories beside the theory offering that are used for investigative reasoning. The quantitative theory should also be reflected in the qualitative theory.

[7] Scales of Transformation – Human-Environmental Interaction in Prehistoric and Archaic Societies: https://www.sfb1266.uni-kiel.de/en

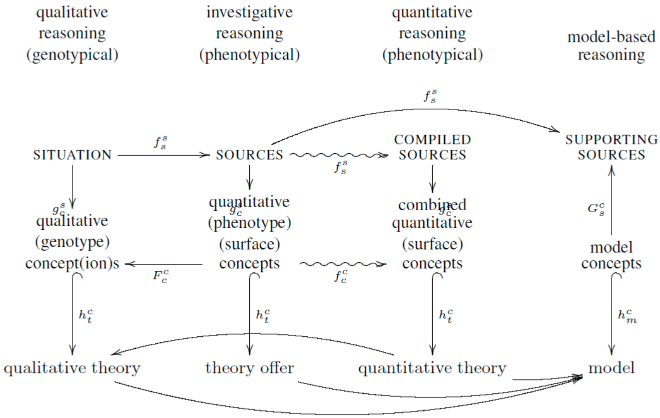

Figure 6: Models for investigative and quantitative reasoning in empirical research

A qualitative theory uses a concept or conception space that represents situations of interest (may be based on some mapping *g* from source data to concepts and some *G* mapping from concepts to sources). The situation can be observed and characterised by sources (may be based on some *f* or *F* mapping between sources). Empirical research in sciences often differentiates between an investigative reasoning and quantitative reasoning. Both use phenotypical observations on proxies. Quantitative approaches aggregate and combine the source data and thus allow to reason on correlation, dependencies, time and spatial relationships. The first two reasoning approaches should be based on a commuting diagram, i.e. the *g*-mapping of a situation equals to the *f*-*g*-*F*-mapping of this situation.

Evidence-based proxy modelling and reasoning treats models in a different way.

(α) Models represent only acceptable possibilities. Each model captures a distinct set of possibilities to which the current description refers) which are consistent with the premises and the knowledge gained so far what makes them intrinsically uncertain because they mirror only some properties they represent.

(β) Models are proxy-driven. The structure of the model corresponds to the proxies it represents.

(γ) Models represent only what has been observed and not what is false in each possibility in contrast to fully explicit models (also representing what is false).

(δ) The more proxies that are considered, and the richer those models are, the more accurate the world view is. (ϵ) Additionally, we use pragmatic reasoning schemata, e.g. A causes B ; B prevents C ; therefore, A prevents C .

The model themselves illustrate then concepts. Therefore, sources support concepts and conceptions what inverts the mapping (G *mapping* instead of g mapping).

Let us now consider the theory-oriented research. The frame for empirical research is similar to communication frames in Subsection 2.5. We neglect inverse modelling [Men89] although it is an important approach to science and it has been reconsidered and generalised under various other names, e.g. [ASG13, Noa09, SV05, BST06, TT13]. Data science approaches have been considered in [KT17].

So, we arrive with the hexagon in Figure 7.

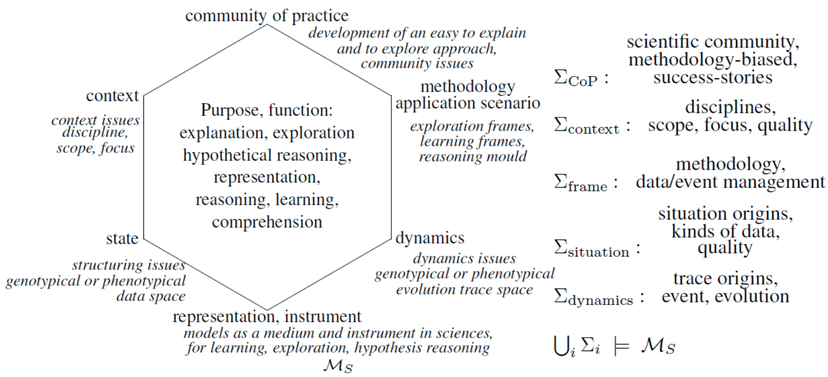

Figure 7: Models are used in natural, social, and other sciences as enhancements and contributions to sciences and as instruments: science contribution, explanation, exploration, learning, comprehension, intellectual absorption, simulation, and reasoning sce- nario

Models function as instruments within the science. They are vehicles for investigation, for analysis, for discovery of alternatives, for prognosis, for exploration, for explanation, for intellectual absorption, for learning, for understanding, for scoped and focused comprehension, for representation of certain aspects, for discussion with partners within their background, for quick illustration, etc. They are supported by various kinds of reasoning. It seems that this variety is rather broad. If we however orient our investigation on the scenarios then we discover that the model utilisation scenarios determine the function of the model. At the same time, the background with the grounding and basis strikes through. Models are biased by their foundations, by their development and utilisation methods, their communities of interest, and their context. A specific context is the school of thought [Bab03, Fle11]. The concept space determines what could the content and the scope of a model. The MMM compendium [TN15] illustrates that models, the approach for to model, and modelling share a good number of common approaches.

2.4. Conceptual Models

Conceptual models are widely used in Computer Science and more specifically in Computer Engineering. In Computer Science and Computer Engineering, one main scenario is (1) the model-based construction of systems beside (2) the explanation and exploration of an application, (3) description of structure and behaviour of systems, and the (4) prognosis of system properties. Model-based construction might include conceptualisation. The application scenarios mainly follows the description-prescription frame. The model is used as a description of its origin and as a prescription of the system to be constructed. The notion of conceptual model is not commonly agreed however[8]. In a nutshell, a *conceptual model* is an language-determined enhancement of a model by concepts from a concept(ion) space.

The conceptual modelling method uses a canonical style of model development and utilisation. Models are instruments in perception and utilisation scenarios. They function is explicitly defined, e.g. models for design and synthesis. The scenario can incorporate a decision point that stops after understanding the perception and domain-situation models or that designs and synthesises the conceptual model after a preparation phase. The last stage support then evaluation and acceptance of the model.

[8] We know almost threescore different notions what shows the wider controversy about this notion[Tha18a]. E.g., Wikiquote (see [Wik17]) lists almost 40 notions. Facetted search for the term “conceptual model” in DBLP results in more than 5.000 hits for titles in papers (normal DBLP search also above 3.400 titles)

So, Figure 8 displays the more specific way of conceptual modelling for information systems. The IS com- munity with its actors shares an IT orientation. It might however be in conflict with the business users. They reason in a different way and are often using a local-as-viewpoint approach. The global-as-design approach might not provide an appropriate support. The model development and utilisation becomes canonical after the choice of the enabling language and the modelling method. The origin models such as the perception and domain-situation models follow the style accepted in these communities. The global-as-design approach must then provide appropri- ate aggregations and derivations for support of local viewpoints. The community also shares the assumption of strict separation of specification into syntax and semantics with the firstness paradigm [KL13, Pei98] for structures and the secondness [Cas55] of functions and views. The model to be developed inherits all the paradigms, assumptions, biases, conceptualisations, cultures, background theories, etc.

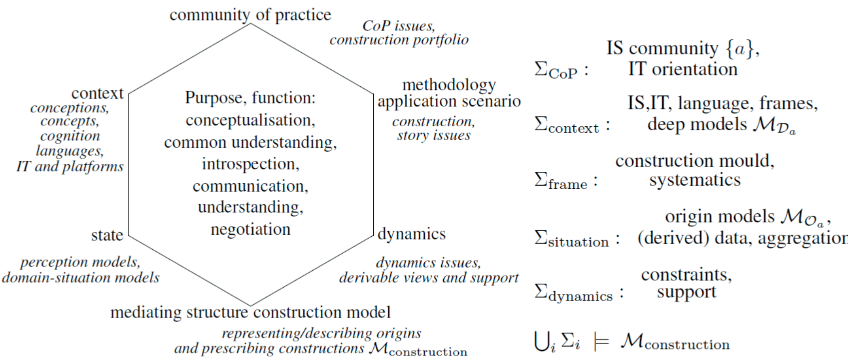

Figure 8: Conceptual models for IS structuring

A typical example for conceptual modelling is entity-relationship modelling. [Tha18b] observed a large number of paradigms, postulates, specific modelling cultures, common-sense, practices, and assumptions such as global-as- design (with derivate ion of local viewpoints), Salami slice typing (for homogenisation of object structure within a class), set semantics (instead of multi-set semantics that is used for implementation), uniqueness of names within a schema, hidden implementation assumptions, specific styles for *model composition one must follow, well- formedness conditions, etc. Some approaches add also requirements such as strict binarisation of all relationship types.*

The notion of conceptualisation, conceptual models, and concepts are far older than considered in Computer Science. The earliest contribution to models and their conceptualisations we are aware of is pre-socratic philosophy and especially the work by Heraclitus [Leb14].

2.5. Models for Communication and Human Interaction

Human communication heavily uses models. They are often not called models. Some models might be metaphors or prototypes. Other models might be incomplete or not really coherent or consistent. They are however used for exchange of opinions among users. Models function in communication scenario as a medium. The communication itself determines the role and thus the function and therefore the purpose of the model. Models represent in this case a common understanding of the communication partners. They are biased by these partners. Communication is based on some common understanding about the topic that is under consideration. Partner have already agreed on some background. They use this agreement within their communication. This agreement is based a common reflection and some common model. This model is taken for granted and not further discussed in communication. So, partners agree on some background or deep model. Typically, deep models [KT17, Tha17b] are not explicitly communicated. We need however an understanding of a theory of deep model and return to it in the next Section. The model is used for a shared understanding, for sense making, for reflection, for derivation of open issues, and for negotiation. Human interaction is typically context-biased and agent-oriented. The same utterance might have many meanings depending on the context, the receiver, and the receiver-sender relationship. A classical easy to understand model is the topographical model of the Königsberg bridge problem [MV10]. The topography abstracts from any non-essential details while adding some abstractions for the identification of the bridges and the teritorries.

The hexagon in Figure 9 shows the differences between models in Mathematical Logics or sciences and communication model. The main difference is the explicit community dependence of such models. Each of the partners or agents *a* has some understanding of the world. This understanding is the main ingredient of a personal model that we call below perception model. The perception model also reflects the setting of the agent, especially the orientation and the priming. The communication might also be based on some common understanding, i.e. on a situation model. The situation model represents the common world view, shared knowledge and beliefs, and shared opinion. The modelling methods is governed by communication and human interaction. So, we might base the frame on the dialogue and interaction frame. Models play a different role. They are used for common understanding. Typical specific models for human interaction are metaphors [Lak87].

Our second case shows the differences and also commonalities between Mathematical Logics and human interaction. The model must suffice all hidden agreements within the community of practice, the context, and the specific scope and focus taken by the agents. Therefore, the logics becomes now more advanced. Mathematical Logic as the opposite is oriented on general laws and thus not oriented on one model but rather on a family of models.

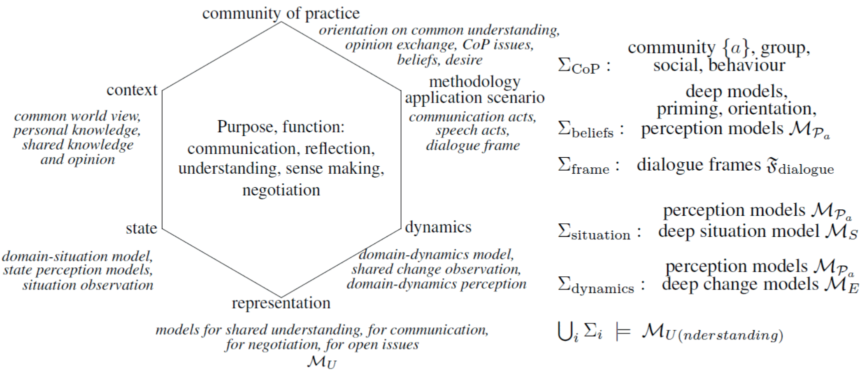

Figure 9: Models in human interaction: development of common understanding, exchange of opinion, communication, reflection, negotiation; context on the basis of commonalities in world views as deep models; scenario based on communication acts

Communication is often based on models that refer to some common understanding of humans. Models in communication might become situation-domain models which describe a common understanding of the phenomena in an area of interest, e.g. structuring of data in a business application case.

2.6. Lessons Learned with the Case Studies

We may now summarise the experience we gained:

- We realise by these case studies that there exists a common framework to models, to the activities of modelling and to modelling as a systematics reflection, for development of models, and for utilisation of models.
- Models are used to represent certain issues. They are more focused and must serve its purpose. The purpose and the focus determine which kind of adequacy is appropriate.
- Models do not exist on their own. They represent something in the world. The world under consideration depends again on the modelling frame. In most cases, mental models and perception or situation models are the origins which are reflected by the model.
- The justification must be given in a way that can be accepted by its community of practice. Models are developed by some members of this community and are utilised by some – may be other – members of this community of practice. So, models must be satisfying. Therefore, we need an explicit understanding of the sufficiency and thus quality of the given model.
- Models are composed of models that reflect their background and of models that represent specific states and situations within from one side and specific dynamics.
- Models are used as instruments in certain scenarios. They have a number of specific functions in these scenarios.
- Models are typically multi-models, i.e. an association of models which are reflecting specific sides of the same issue depending on the viewpoint that is actually considered. Since such models must be coherent we may bundle them within a model suites [DT10, Tha10].

- Model development and model utilisation typically follow canonical stories. An example is mathematical modelling that consists of a six-step procedure. similar procedures can be observed for most sciences that start with a research question, initialise a certain research agenda or problem solving program or schedule, adapt elements to be used to this program, and then solve a problem. Solution-faithfulness is assumed as a hidden quality characteristics beyond the problem invariance. Modelling is typically based on some specific method or methodology, e.g. the mathematical method. These methods are a mould for the modelling process itself, e.g. a pattern, template, stereotype, work-holding attachment, and an appliance. The method itself follows a macro-model.
- Modelling is still a big challenge to science and has a lot of lacunas. The biggest lacunas seem to be the missing support for combined model-based reasoning. Conceptual modelling uses a specific kind of layered model-based reasoning with changing reasoning methods depending on the stage of model development and model utilisation, e.g. in greenfield development of conceptual models: settlement of the context and the method, transfer of mentalistic concepts to codified ones with a concept expression language, transfer of domain-situation models to raw conceptual models, language-backed negotiation and agreement on a number of conceptual models that allow reflexion of different viewpoints, maturation of these conceptual models, and proper documentation. The reasoning method changes according to the stages. The integration of all these reasoning methods into a holistic one is not required.

3. Towards a General Theory of Models

3.1. Deep Models and the Modelling Matrix

The context and methodology layer determines the set-up of the model. It is often taken for granted and as given. It makes modelling more economical and also more reliable. A number of quality characteristics can be thus satisfied without any further consideration. Model development is typically based on an explicit and rather quick description of the 'surface' or normal model and on the mostly unconditional acceptance of this set-up. In reality, this setting becomes an essential part of the model. We call it deep model [Tha18b]. It directs the modelling process and the surface or normal model. Modelling itself is often understood as development and design of the normal model. This approach has the advantage that the deep model can be used as a basis for many models.

The set-up is the modelling matrix behind the model. It consists of the grounding for modelling (paradigms, postulates, restrictions, theories, culture, foundations, conventions, authorities), the outer directives (context and community of practice), and basis (assumptions, general concept space, practices, language as carrier, thought community and thought style, methodology, pattern, routines, commonsense) of modelling. It uses a collection of undisputable elements of the background as grounding and additionally a disputable and adjustable basis which is commonly accepted in the given context by the community of practice.

The modelling matrix is often given as a stereotype one should follow while developing the normal model. Adequacy and dependability of is partially already defined by such stereotypes. The stereotype of a modelling process is based on a general modelling situation.

Stereotypes determine the model kind, the background and way of modelling activities. They persuade the activities of modelling.

3.2. The Five Concerns and a General Approach to Modelling

The case studies led us to the conclusion that there is a common three-layer setting in modelling:

(1) **Community and scenario setting:** The community governs the function that a model has to serve according to their issues and scenario.

Community of practice and application cases: The community of practice has its needs and desires. It faces a number of application cases. The application case consists of tasks that should be accomplished. These tasks form the community portfolio. The application cases can be solved by a model, i.e. the model functions as an instrument. Community members determine which model functions best. The community agrees on the issues for modelling.

(2) **Guiding settings:** The deep model and the matrix is commonly agreed according to the setting in the first layer.
Context: Modelling has its implicit and sometimes also its explicit context. Knowledge and disciplinary schools of thought and understanding are considered to be fixed. In a similar form, the background is fixed. This context forms the deep model that underpins the entire modelling process. A typical element of the deep model is the school of thought.
Modelling methodology and application mould: Modelling follows typically practices that are accepted within the community of practice. These practices are often stereotyped. The methods that are used for model development

(3) **Origins and targets:** Members of the community form their personal perception models and share their domain- situation model that characterises states and dynamics in the application domain that is of interest. These models are the origins on which the normal model is formed as an extension of the deep model.

The final result is a model that combines the normal and the deep models. The representation of the final model must not show all details of the deep model.

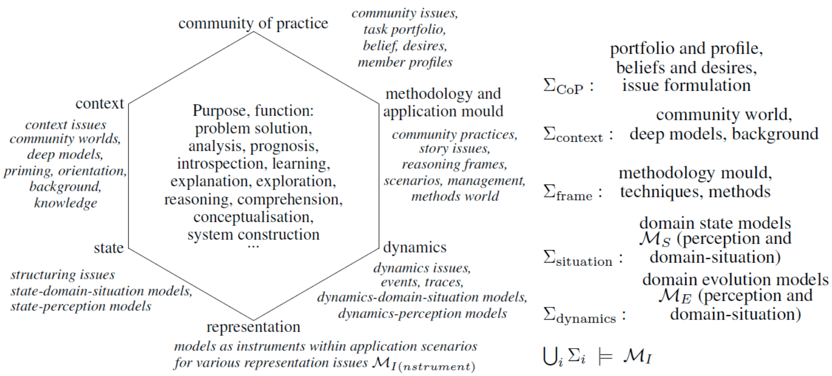

Figure 10: The five concerns for models as a kernel for a theory of models and of modelling

This general setting takes us back to the rhetorical frame[9] and its generalisation to the W∗H specification frame- work [DT15]: In our case, the model ("what") incorporates the meaning of parties (semantical space; "who") during a discourse ('when') within some application with some purpose ("why") based on some modelling language.

We thus distinguish between *five grounding and driving perspectives* to models:

- *Community perspective:* The community has intentionally set-up its application cases, its interests, its desires and its portfolio. The community communicates, knows languages, explains, recognizes, accept the grounding behind the models, has been introduced to the basis and is common with it. Models are used by, developed by and for, and gain a surplus value for a community of practice. They may have a different shape, form, and value for community members. They must, however, be acceptable for its community. Typical specialisations of this concern are 'by whom', 'to whom', 'whichever', and 'worthiness'.
- *Purpose, function, goal perspective:* Models and model development serve a certain purpose in some utilisation scenarios. The model has to function in these scenarios and should thus be of certain quality. At the same time it is embedded into the context and is acceptable by a community of practice with its rules and understandings. We answer 'why' and 'for which reason' questions.
- *Product perspective:* Models are products that are requested, have been developed, are delivered according to the first perspective, are potentially applicable within the scenarios, and have their merits and problems. Typ- ical purpose characteristics are answers to 'how-to-use', 'why', 'whereto', 'when', for which reason" and
- 'wherewith' (carrier, e.g., language) questions.
- *Engineering perspective:* Models are mastered within an engineering process based on some approaches to mod- elling activities and to uti-

[9] It relates back to Hermagoras of Temnos or Cicero more than 2000 years ago., i.e. they are characterised through "who says what, when, where, why, in what way, by what means" (Quis, quid, quando, ubi, cur, quem ad modum, quibus adminiculis). The Zachman frame used in software engineering and computer science is only at its best a reinvention of this frame.

lisation of models. Modelling is a systematically performed process that uses meth- ods, techniques, preparations, and experience already gained in former modelling processes. The modelling method is typically given in a canonical form. It guides and steers the model development and the model utilisation processes. This guidance can be derived from the scenarios in which the model functions.

- *Background and context perspective*: Model development and utilisation is a systematic, well-founded process that allows one to reason on the capacity and potential of the model, to handle adequacy and dependability of models in a proper way, and the reason on the model and its origins that it represents. A modelling culture also answers the by-what-means question beside providing the background. The background is typically con- sidered to be given and not explicitly explained. It consists of an undisputable grounding and of a disputable and adjustable basis. The context clarifies on which basis and especially on which grounding the model has been developed and must be restricted in its utilisation. Additional context characteristics are answers to questions about the 'whereat', 'whereabout', 'whither', and 'when'.

Models are typically laden by these grounding and driven perspective. They are the hidden part of the deep model. In daily practice, modelling is mainly modelling `beside or additional' to the deep model. The modelling matrix is also taken for granted. That means, modelling is mainly normal modelling that incorporates and unconditionally accepts the perspectives.

3.3. Model-Based Reasoning

The observation depicted in Figure 6 drives us to a multi-model approach. We build models in situations, concepts and theories in dependence on their function and purpose. The same situation-concept-theory may be the basis for a variety of models. A typical multi-model approach is the consideration of models in Physics. Models should thus be considered to be the third dimension of science [BFA+ 16, TN15, TTF16]. Disciplines and also human communication, human interaction, and human collaboration have developed a different understanding of the notion of model, of the function of models in scientific research and communication. Models are often considered to be artifacts. In reality, they are however instruments that are used with a certain intention. Models might also be perception models that incorporate mentalistic concepts [Jac04]. Models are used in various *utilisation scenarios* such as construction of systems, verification, optimization, explanation, and documentation. *In these scenarios* they *function* as *instruments* and thus satisfy a number of properties.

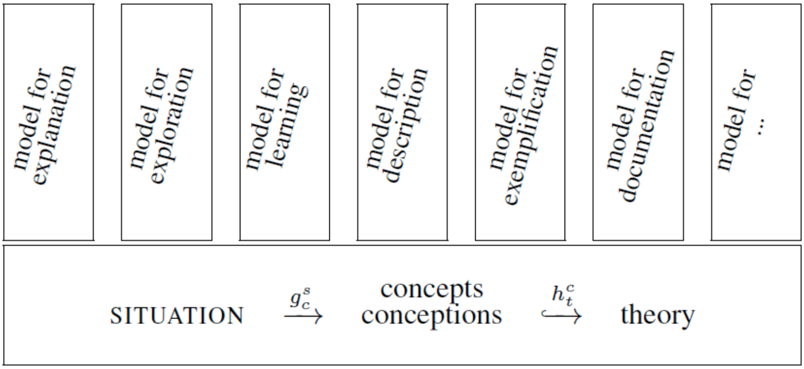

Figure 11: Models as specific representations of situations, concept(ion)s, and theories

Model-based reasoning [Bre10, Mag14] is enhances classical reasoning such as reasoning mathematical calculi or logical derivation. There are several kinds of reasoning that are more appropriate and widely used:

Evidence-based modelling and reasoning is one of the main approaches for quantitative models. Models only represent acceptable possibilities. Each model captures a distinct set of possibilities to which the current de- scription refers. Possibilities are consistent with the premises and the knowledge gained so far what makes them intrinsically uncertain because they mirror only some properties they represent. In investigative and quantitative modelling, models can be proxy-driven where the the structure of the model corresponds to the proxies it represents. They might also include abstractions such as negation which must be then stratified. Propositional evidence-based reasoning is based on monotone functions and specific interpretations for log- ical connectives. Models represent in this case only what has been observed and not what is false in each possibility what is different from fully explicit models which represent what is false. The more proxies are considered the richer those models are, the more accurate the world view is. Evidence-based modelling and reasoning uses pragmatic reasoning schemata, e.g. A causes B; B prevents C; therefore, A prevents C. The calculus may use several implication forms, e.g. deterministic conclusions (A cause B to occur: given A then B occurs) and ordered sets of possibilities (A enables B to occur: given A then it is possible for B to occur).

Hypothetical and investigative modelling considers different assumptions in order to see what follows from them, i.e. reasons about alternative possible worlds (i.e. states of the world), regardless of their resemblance to the actual world. Potential assumptions with their possible world conclusions and assertions are supported by a number of hypotheses (allowing to derive them). It is often combined with abductive reasoning. Evidence against hypothesis is performed by testing its logical consequences, i.e. exploring different alternative solu- tions in parallel to determine which approach or series of steps best solves a particular problem.

Causal reasoning and modelling is a specific variant of inductive reasoning and justification-backed truth main- tenance with assertions (beliefs, background) and justifications within some context (current beliefs, justifications, arguments). It establishes the presence of causal relationships among events based on methods of agreement, difference, concomitant variation, and residues. It uses assumptions and thus avoids inconsistent sets ('nogood' environment). The environment consists of a set of assumptions, premises, assumed state- ments, and derived statements for the world view. Justifications (e.g. data-supported) represent cause. Hypotheses are not derived from evidence but are added to evidence. They direct the search for evidence. They are tested by modus tollens ($(H \rightarrow I) \wedge \neg I \Rightarrow \neg H$).

Network reasoning uses models that are expressed as networks. Nodes carry justification (arguments) and status (in, out, believed, relevant, necessary, …). Edges, hyperedges, or directed edges have an antecedent (support nodes) and conclusions. They may also be non-monotonic and enable backtracking for dependencies (causal- ity, chronological, space, etc. Labels also express the degree of consistency and believability. Queries can be expressed as subgraphs and are evaluated by query embedding into the network.

Model-based reasoning is an interactive and iterative process that helps to digest a theory and to develop the theory. Therefore, model-based reasoning integrates many reasoning approaches, e.g. deduction, induction, abduction, Solomonoff induction, non-monotonic reasoning, and restrospective reasoning. Model refinement might also be based on inverse modelling approaches. Facets of the last one are inductive learning, data mining, data analysis, generic modelling, universal applications, systematic modelling, and pattern-based reasoning.

3.4. Towards Powerful Methodological Moulds

The hexagon picture and the consideration of the variety of different (reasoning) techniques might lead to the impression that a general treatment of models and a methodological support is infeasible. Sciences and humans have however developed their specific approaches and overcome the challenges of this complexity. We will illustrate resolution of complexity by two methods: Layered treatment and generic modelling. Both approaches are based on the separation of a model into a deep or core model and a normal model. A typical example of a methodology is the mathematical modelling method [BT15, GKBF13, vDGOG09, Pol45] (see Subsection 2.2). The CRISP cycle (data selection according to generic model, data preprocessing, data transformation, data mining, model development, interpretation/evaluation) [BBHK10] and classical investigation cycles (define issues and functions of the model, hypothetically predict model properties, experiment, (re)define model, apply and validate the model against the situation) are typical methodologies. Similar methodologies are known for data mining [Jan17], data analysis [BBHK10], and systematic mathematical problem solving [Pod01]. They use a variety of reasoning techniques and layer their application of these techniques according to the stage that is currently under consideration. These modelling methods and methodologies are used similar to moulds that are commonly used in manufacturing.

Data mining [Jan17], inverse modelling [RSS+ 10], and generic modelling [TTFZ14] start with a generic model.

A set of associated models (called model suite) is the result of a modelling process. We may develop a singleton model or a model suite. Figure 12 displays a variant that starts with an initialisation and setting of the modelling process. The initialisation is based on the issues that are important for the community of practice, the tasks that are on the agenda, and the injection of the context. The community of practice aims at completion of tasks from its portfolio and is bound by profiles of their members what also includes beliefs and desires shared in this community. At the same time, the methodology for modelling is already chosen. That means, the upper dimensions in Figure 10 governs the entire modelling process. A similar approach can be declared for model redevelopment model evolution instead of model development from scratch (greenfield modelling). The result of the first layer is a deep model and a matrix.

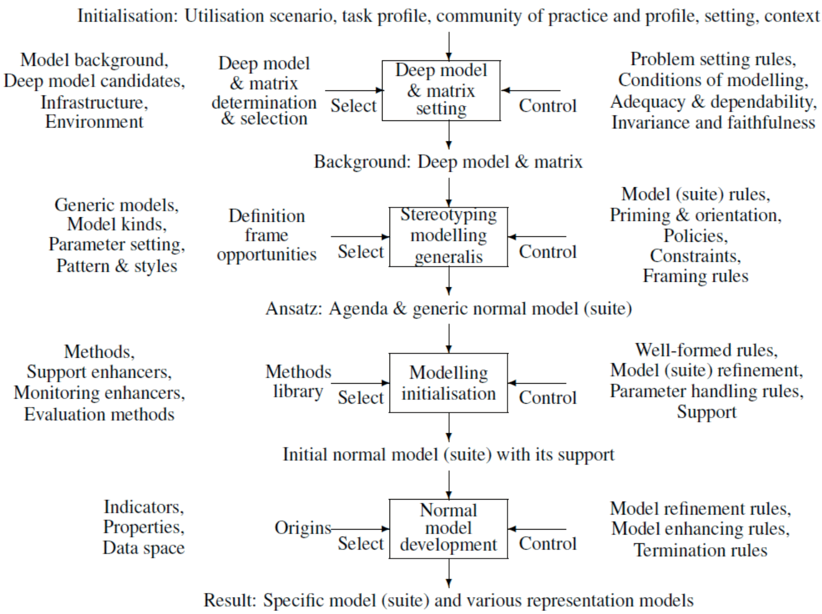

Figure 12: Layered model (suite) development (None-iterative form, greenfield variant)

The second layer or stage uses some kind of most general and refinable model as the initial model. A generic model [BST06, TF16] is a general model which can be used for the function within a given utilisation scenario and which is not optimally adapted to some specific origin collection. It is tailored in next steps to suit the particular purpose and function. It generally represents many origins under interest, provides means to establish adequacy and dependability of the model, and establishes focus and scope of the model. Modelling is often based on some experience. This experience can be systematically collected within a number of libraries. Libraries and collections are used for collecting the most appropriate setting and model. This selection is controlled or governed by rules, restrictions, conditions, and properties. The main results of the second layer are generic models and an agenda for the next modelling steps.

The third layer sets the environment for the development of the normal model. This environment prepares model development on the basis of the generic models and under inclusion of the deep model. The section of methods might also include the selection of parts and pieces from the context, e.g. from the background and especially from theories and knowledge. The fourth layer results then in the development of a normal model that can be neatly combined with the deep model. Representation models are developed for different members of the community of practice and for different functions the model must fulfill in the utilisation scenario.

This development process is often cut down to the fourth layer assuming the results of the first, second, and third layer as already given. This kind of implicitness has often been assumed for language utterance. The government and binding approach [Cho82, BST06] made the two-step generation of sentences explicit: we intentionally prepare the deep model and then express ourselves by an explicit statement which is build similar to a combination of a normal model and of a cutout of the deep model.

4. Conclusion

A collection of modelling approaches has been presented in [TN15]. It seems that the variety of modelling approaches, the different utilisation of model, the broad span of underpinning theories, the variety of models themselves do not allow to develop a common setting for models. We often met the claim that models used in social and natural sciences, in mathematics, in logics and in daily life are so different that a common treatment cannot exist. From the first side, logicians provided a specific understanding of models that is easy and formally to handle. They inspired model research and the notion of model, e.g. [Bal16, Kas03, Mah09, Mah15, Sta73, Ste66, Ste93]. This notion has mainly been based on properties that a model should satisfy: mapping, truncation, and pragmatic properties as phenomenalistic characterisation of the notion. From the second side, models in all sciences have been used as an artifact for solution of problems, e.g. [BT15, Her84, vDGOG09, vN55]. The model notion has been enhanced by amplification, distortion, idealisation, carrier, added value, and purpose-preservation properties. From the third side, language- and concept-based foundations of models have been developed in philosophy of science and linguistics [Bla¨15, Bur15, Cas55, KL13, Lat15, Pei98]. From the fourth side, models in engineering [BFA+ 16, LH15, TD16, TTF16] are instruments for system construction. From the sixth side, models are also in- struments in human interaction. They are used as metaphors, for communication, for brief reference, for depiction, as prototype, etc. For instance, the question whether a picture or a photo is a model depends on their utilisation in some interaction scenarios. We thus may conclude that a common science and culture of modelling cannot exist.

The main claim in this paper is however that a common treatment of models in science and human interaction can be developed. We base our foundational framework on a separation of concern. This separation into five governors for models provides a common treatment of models and model utilisation. We base our framework on the observation that not all concerns are considered at the same time. So, we can use some kind of stepwise procedure for model development.

Utilisation of models as instruments in scenarios is the main driving property that distinguishes something from a model. The model functions in scenarios such as communication, reflection, understanding, negotiation, expla- nation, exploration, learning, introspection, theory development, documentation, illustration, analysis, construction, description, and prescription.

How the model functions has been illustrated in the case of model-based reasoning. Model-based reasoning goes far beyond model methods used in classical first-order predicate logics or mathematics. We use the layering approach also for model methods since the development of a general reasoning method is far beyond the horizon.

The meta-models of modelling concerns in Figures 3, 4, 7, 8, 9, 10 support the layered modelling method in Figure 12. Instead, we could separate the layers into communities and their application scenario, into background and methodology setting, into situation and theory setting, into origin calibration, and model delivery layers.

This paper has been centered around models, theories, communities, context, methodologies, state, and dynamics at the same level of abstraction. Model-driven development and architecture [MMR+ 17, SV05] is an orthogonal approach to this paper. It distinguishes abstraction layers for models (M1), model frames (M2) [as meta-models], model frameworks (M3) [as meta-meta-models], and model framework setting (M4). The data/information and traces/events abstraction layer (M0) underpins models. Our approach has been mainly oriented on M1. We envision that the general M0-M1-M2-M3-M4 architecture can be integrated into our approach as well.

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